Integrals

1. $\int \sin\left(\frac{1}{3}x\right)dx$

$\boxed{-3\cos\left(\frac{1}{3}x\right)+C}$

2. $\int \frac{\sqrt{\ln x}}{x}dx \quad u=\ln x$

$\int u^{1/2}du \quad du=\frac{1}{x}dx$

$\boxed{\frac{2}{3}(\ln x)^{3/2}+C}$

3. $\int \frac{4}{2-x}dx$

$\boxed{-4\ln(2-x)+C}$

4. $\int e^{\cos x}\sin x\,dx \quad u=\cos x$

$-\int e^u du \quad du=-\sin x\,dx$

$\boxed{-e^{\cos x}+C} \quad -du=\sin x\,dx$

5. $\int \frac{\sec^2 x}{\tan x+3}dx \quad u=\tan x+3$

$\int \frac{1}{u}du \quad du=\sec^2 x\,dx$

$\ln|u|+C$

$\boxed{\ln|\tan x+3|+C}$

6. $\int\left(\frac{x^3}{7}+x^5\sqrt{x}\right)dx$

$\int\left(\frac{x^3}{7}+x^5\cdot x^{\frac{1}{2}}\right)dx$

$\int\left(\frac{x^3}{7}+x^{11/2}\right)dx$

$\boxed{\frac{x^4}{28}+\frac{2}{13}x^{13/2}+C}$

7. $\int x^3(1+3x^4)^2dx \quad u=1+3x^4$

$\frac{1}{12}\int u^2du \quad du=12x^3dx$

$\frac{1}{12}\cdot\frac{1}{3}u^3+C \quad \frac{1}{12}du=x^3dx$

$\boxed{\frac{1}{36}(1+3x^4)^3+C}$

8. $\int_{-1}^{1} x\sqrt{1-x^2}dx \quad u=1-x^2$ $\quad 1-(-1)^2=0$, $1-(1)^2=0$

$-\frac{1}{2}\int_0^0 u^{1/2}du \quad du=-2x\,dx$

$\boxed{0} \quad -\frac{1}{2}du=x\,dx$

9. $\int \csc^2(4x)dx$

$\boxed{-\frac{1}{4}\cot(4x)+C}$

10. $\int \frac{\sqrt[4]{\ln x}}{x}dx \quad u=\ln x$

$\int u^{1/4}du \quad du=\frac{1}{x}dx$

$\frac{4}{5}u^{5/4}+C$

$\boxed{\frac{4}{5}(\ln x)^{5/4}+C}$

11. $\int \frac{7}{x-3}dx$

$\boxed{7\ln|x-3|+C}$

12. $\int x^4\sqrt{x}\,dx$

$\int x^{9/2}dx$

$\boxed{\frac{2}{11}x^{11/2}+C}$

13. $\int \frac{\sec x\tan x}{\sec x-5}dx \quad u=\sec x-5$

$\int \frac{1}{u}du \quad du=\sec x\tan x\,dx$

$\ln|u|+C$

$\boxed{\ln|\sec x-5|+C}$

14. $\int \frac{1}{e^{3x}}dx$

$\int e^{-3x}dx$

$\boxed{-\frac{1}{3}e^{-3x}+C}$

15. $\int \frac{x^5-2x}{x^4}\,dx$

$\int x - 2x^{-3}\,dx$

$\boxed{\frac{1}{2}x^2 + x^{-2} + C}$

16. $\int_0^1 x^5(3-x^6)^4\,dx$ $u = 3-x^6$ $3-(0)^6=3$ $3-(1)^6=2$

$-\frac{1}{6}\int_3^2 u^4\,du$ $du = -6x^5\,dx$

$-\frac{1}{6}\cdot\frac{1}{5}u^5\Big|_3^2$ $-\frac{1}{6}du = x^5\,dx$

$\boxed{-\frac{1}{30}[2^5-3^5] \text{ OR } \frac{1}{30}[3^5-2^5]}$

17. $\int \sin 9x\,dx$

$\boxed{-\frac{1}{9}\cos(9x)+C}$

18. $\int \frac{\ln^8 x}{x}\,dx$ $u = \ln x$

$\int u^8\,du$ $du = \frac{1}{x}dx$

$\frac{1}{9}u^9 + C$

$\boxed{\frac{1}{9}(\ln x)^9 + C}$

19. $\int \frac{7}{x+21}\,dx$

$\boxed{7\ln|x+21| + C}$

20. $\int \frac{1}{e^{7x}}\,dx$

$\int e^{-7x}\,dx$

$\boxed{-\frac{1}{7}e^{-7x} + C}$

21. $\int \frac{\csc^2 x}{\cot x - 15}\,dx$ $u = \cot x - 15$

$\int -\frac{1}{u}\,du$ $du = -\csc^2 x\,dx$

$-\ln|u| + C$ $-du = \csc^2 x\,dx$

$\boxed{-\ln|\cot x - 15| + C}$

22. $\int \frac{5x^3+3x-4}{x^2}\,dx$

$\int \left(5x + \frac{3}{x} - \frac{4}{x^2}\right)dx$

$\boxed{\frac{5}{2}x^2 + 3\ln|x| + 4x^{-1} + C}$

23. $\int x^2\sqrt{x^3+3}\,dx$ $u = x^3+3$

$\frac{1}{3}\int u^{1/2}\,du$ $du = 3x^2\,dx$

$\frac{1}{3}\cdot\frac{2}{3}u^{3/2} + C$ $\frac{1}{3}du = x^2\,dx$

$\boxed{\frac{2}{9}(x^3+3)^{3/2} + C}$

24. $\int_{-1}^{1} x^3(1+x^4)^3\,dx$ $u = 1+x^4$ $1+(-1)^4=2$ $1+(1)^4=2$

$\frac{1}{3}\int_2^2 u^3\,du$ $du = 3x^3\,dx$

$\boxed{0}$ $\frac{1}{3}du = x^3\,dx$

25. $\int \frac{\csc x\cot x}{\csc x + 3}\,dx$ $u = \csc x + 3$

$\int \frac{1}{u}\,du$ $du = -\csc x\cot x\,dx$

$-\ln|u| + C$ $-du = \csc x\cot x\,dx$

$\boxed{-\ln|\csc x + 3| + C}$

26. $\int \tan x\,dx$

$\int \frac{\sin x}{\cos x}\,dx$ $u = \cos x$

$-\int \frac{1}{u}\,du$ $du = -\sin x\,dx$

$-\ln|u| + C$ $-du = \sin x\,dx$

$\boxed{-\ln|\cos x| + C}$

27. $\int x\sqrt{x}\,dx$

$\int x^{3/2}\,dx$

$\boxed{\frac{2}{5}x^{5/2} + C}$

28. $\int (e^{4x} - \sin(3x))dx$

$\boxed{\frac{1}{4}e^{4x} + \frac{1}{3}\cos(3x) + C}$

29. $\int e^{\sin x}\cos x\,dx \quad u = \sin x$

$\int e^u du \quad du = \cos x\,dx$

$e^u + C$

$\boxed{e^{\sin x} + C}$

30. $\int_0^1 \sqrt{x^5+2x}'(5x^4+2)dx$

$\int_0^3 u^{1/2}du \quad u = x^5+2x$

$\frac{2}{3}u^{3/2}\Big|_0^3 \quad du = 5x^4+2$

$\boxed{\frac{2}{3}(3)^{3/2}}$ $\quad 0^5+2(0)=0$, $1^5+2(1)=3$

31. $\int \frac{\ln^9 x}{x}dx \quad u = \ln x$

$\int u^9 du \quad du = \frac{1}{x}dx$

$\frac{u^{10}}{10} + C$

$\boxed{\frac{1}{10}\ln^{10}x + C}$

32. $\int \frac{3\cos 5x}{\sin 5x}dx \quad u = \sin 5x$

$\frac{3}{5}\int \frac{1}{u}du \quad du = 5\cos 5x\,dx$

$\frac{3}{5}\ln|u| + C \quad \frac{3}{5}du = 3\cos 5x\,dx$

$\boxed{\frac{3}{5}\ln|\sin 5x| + C}$

33. $\int \cos(4x+5)dx$

$\boxed{\frac{1}{4}\sin(4x+5) + C}$

34. $\int \frac{\ln x}{5x}dx \quad u = \ln x$

$\frac{1}{5}\int u\,du \quad du = \frac{1}{x}dx$

$\frac{1}{5}\cdot\frac{u^2}{2} + C$

$\boxed{\frac{1}{10}(\ln x)^2 + C}$

35. $\int \frac{3}{7-x}dx \quad u = 7-x$

$-3\int \frac{1}{u}du \quad du = -dx$

$-3\ln|u| + C \quad -du = dx$

$\boxed{-3\ln|7-x| + C}$

36. $\int \sin x e^{\cos x}dx \quad u = \cos x$

$-\int e^u du \quad du = -\sin x\,dx$

$-e^u + C \quad -du = \sin x\,dx$

$\boxed{-e^{\cos x} + C}$

37. $\int \cot x\,dx$

$\int \frac{\cos x}{\sin x}dx \quad u = \sin x$

$\int \frac{1}{u}du \quad du = \cos x\,dx$

$\boxed{\ln|\sin x| + C}$

38. $\int \frac{x^2+x-1}{x}dx$

$\int x + 1 - \frac{1}{x}dx$

$\boxed{\frac{x^2}{2} + x - \ln|x| + C}$

39. $\int \frac{x+3}{(x^2+6x-5)^2}dx \quad u = x^2+6x-5$

$\frac{1}{2}\int \frac{1}{u^2}du \quad du = (2x+6)dx$

$\frac{1}{2}\int u^{-2}du \quad \frac{1}{2}du = (x+3)dx$

$\boxed{-\frac{1}{2}(x^2+6x-5)^{-1} + C}$

40. $\int_0^1 x\sqrt{1-x^2}\,dx \quad u = 1-x^2$

$-\frac{1}{2}\int_1^0 u^{1/2}du \quad du = -2x\,dx$

$-\frac{1}{2}\cdot\frac{2}{3}u^{3/2}\Big|_1^0 \quad -\frac{1}{2}du = dx$

$-\frac{1}{3}\left[0^{3/2} - 1^{3/2}\right]$ $\quad 1-0^2=1$, $1-1^2=0$

$-\frac{1}{3}(-1) = \boxed{\frac{1}{3}}$