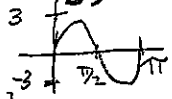
Review for ~~Sec 7.1~~ Chapter 8

The function **v(t)** is the velocity of an object moving on the x-axis at any time $t \geq 0$ where the velocity is measured in ft/sec.

a) Determine when the object is moving right, left and stopped
b) Determine the *acceleration* of the object at t = 2.
c) Determine the *average acceleration* of the object over the given time interval *(set-up only)*
d) Determine the *displacement* of the object over the given time interval
e) Determine the *average velocity* of the object over the given time interval *(set-up only)*
f) Determine the *total distance* traveled by the object over the given time interval. (set-up only)



*do star problems **without** a calculator*

2nd

***1)** $v(t) = e^t - 2$ $[0,4]$

a) right, left, stopped

$v(t) = 0$
$0 = e^t - 2$
$e^t = 2$
$\ln e^t = \ln 2$
$t = \ln 2$

v(t): $-$ on $[0, \ln 2)$, $+$ on $(\ln 2, 4]$

$\boxed{\begin{array}{l}\text{Stopped: } t = \ln 2 \text{ sec, b/c } v = 0 \\ \text{right: } (\ln 2, 4] \text{ b/c } v > 0 \\ \text{left: } [0, \ln 2) \text{ b/c } v < 0\end{array}}$

b) accel. @ t = 2

$v'(t) = a(t) = e^t$
$\boxed{a(2) = e^2 \text{ ft/sec}^2}$

c) ave. acc on [0,4]

$\frac{v(4) - v(0)}{4 - 0} = \boxed{\frac{(e^4 - 2) - (e^0 - 2)}{4} \text{ ft/sec}^2}$

d) displacement on [0,4]

$\int_0^4 (e^t - 2)\,dt$
$= e^t - 2t \Big|_0^4$
$= [e^4 - 2(4)] - [e^0 - 2(0)]$
$\boxed{= e^4 - 9 \text{ ft.}}$

$\int_a^b v\,dt$
$= s \Big|_a^b$
$= s(b) - s(a)$

e) ave. velocity on [0,4]

$\frac{1}{4-0} \int_0^4 (e^t - 2)\,dt$
$\boxed{\frac{1}{4}(e^4 - 9) \text{ ft/sec}}$

f) total distance

$\int_0^4 |v(t)|\,dt$

total $= -\int_0^{\ln 2} (e^t - 2)\,dt + \int_{\ln 2}^4 (e^t - 2)\,dt$
$\boxed{= -11 + e^4 + 4\ln 2 \text{ ft.}}$

1st

*** 2)** $v(t) = 3\sin(2t)$ $[0, \pi]$

a) right, left, stopped

$3\sin(2t) = 0$
$\sin(2t) = 0$
$2t = 0, \pi, 2\pi$
$t = 0, \pi/2, \pi$

v(t): $+$ on $(0, \pi/2)$, $-$ on $(\pi/2, \pi)$

$\boxed{\begin{array}{l}\text{stopped: } t = 0, \pi/2, \pi \text{ sec. b/c } v = 0 \\ \text{right: } (0, \pi/2) \ v > 0 \\ \text{left: } (\pi/2, \pi) \ v < 0\end{array}}$

b) accel. @ t = 2

$v'(t) = a(t) = 6\cos(2t)$
$\boxed{a(2) = 6\cos(4) \text{ ft/sec}^2}$

c) ave. acc on $[0, \pi]$

$\boxed{\frac{3\sin(2\pi) - 3\sin(0)}{\pi - 0} \text{ ft/sec}^2}$

d) Displacement on $[0, \pi]$

$\int_0^{\pi} 3\sin(2t)\,dt$
$= -\frac{3}{2}\cos(2t) \Big|_0^{\pi}$
$= \left[-\frac{3}{2}\cos(2\pi)\right] - \left[-\frac{3}{2}\cos(0)\right]$
$= -\frac{3}{2} + \frac{3}{2}$
$\boxed{= 0 \text{ ft.}}$

e) ave velocity on $[0, \pi]$

$\frac{1}{\pi - 0} \int_0^{\pi} 3\sin(2t)\,dt$
$= \frac{1}{\pi}(0) \text{ ft/sec}$

f) total distance

$\int_0^{\pi/2} 3\sin(2t)\,dt - \int_{\pi/2}^{\pi} 3\sin(2t)\,dt$
$= {}^{*}\,2\int_0^{\pi/2} 3\sin(2t)\,dt$

3rd on next sheet

~~c 3)~~ $v(t) = t^2 - 3t - 4 \qquad [0,5]$

c 4) $v(t) = \sin x(\cos(2x)) \qquad [0,\pi]$
(if there is time)

a) right, left, stopped

$t^2 - 3t - 4 = 0$

$t = \cancel{-1}, 4$

v(t): $-$ on $[0,4]$, $+$ on $[4,5]$

$\boxed{\begin{array}{l} \text{a) stopped: } t = 4 \text{ sec. b/c } v = 0 \\ \text{right: } (4,5] \text{ b/c } v > 0 \\ \text{left: } [0,4) \text{ b/c } v < 0 \end{array}}$

b) accel at $t = 2$

$v'(t) = a(t) = 2t - 3$

$\boxed{a(2) = 2(2) - 3 = 1 \text{ ft/sec}^2}$

c) ave accel on $[0,5]$

$$\frac{[(5)^2 - 3(5) - 4] - [(0)^2 - 3(0) - 4]}{5 - 0} \text{ ft/sec}^2$$

d) displacement on $[0,5]$

$$\int_0^5 t^2 - 3t - 4 \, dt$$

$\boxed{\approx -15.8\overline{3} \text{ ft}}$

e) ave velocity on $[0,5]$

$$\frac{1}{5-0} \int_0^5 t^2 - 3t - 4 \, dt$$

$\boxed{\approx \frac{1}{5}(-15.8\overline{3}) \text{ ft/sec}}$

f) total distance traveled $[0,5]$

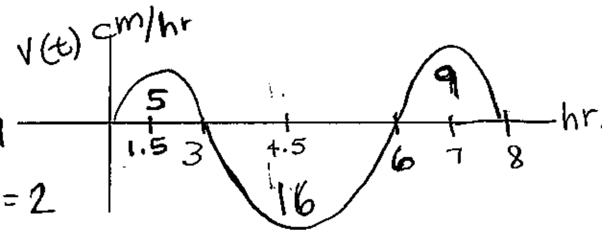
$$\int_0^5 |t^2 - 3t - 4| \, dt$$

$\boxed{\begin{array}{l} \approx 21.499 \text{ ft} \\ 21.500 \end{array}}$

↑ must write zeros

5) A particle moves along a y-axis. Its velocity, measured in cm/hr, is given in the graph below and initial position at $t = 0$ is $y(0) = 4$. The values in each region represent the area of the region.

a) Find the position at each time:



t = 3 $\quad y(3) = 4 + \int_0^3 v(t)\,dt = 4 + 5 = 9$

t = 6 $\quad y(6) = 4 + \int_0^6 v(t)\,dt = 4 + 5 - 16 = -7$

t = 8 $\quad y(8) = 4 + \int_0^8 v(t)\,dt = 4 + 5 - 16 + 9 = 2$

b) What is the displacement of the particle for the first 8 hours?

$$\int_0^8 v(t)\,dt = 5 - 16 + 9 = -2 \text{ cm.}$$

c) What is the displacement by the object for $3 \le t \le 8$

$$\int_3^8 v(t)\,dt = 16 - 9 = -7 \text{ cm.}$$

c) What is the total distance traveled on $[0,8]$?

$$\int_0^8 |v(t)|\,dt = 5 + 16 + 9 = 30 \text{ cm.}$$

d) At what time t is the speed the greatest?

Speed $= |v(t)| \qquad t = 4.5$ hr.

e) When does acceleration equal 0?

$a(t) = v'(t)$

$a(t) = 0$ @ $t = 1.5, 4.5, 7$ hr.

4) $V(t) = \sin t(\cos(2t))$ $[0, \pi]$

a) right, left, stopped

*Anticipate when you think $V(t)=0$

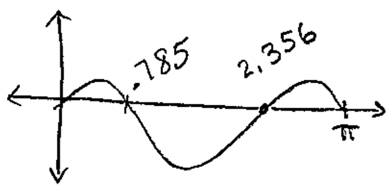
$$0 \cdot \text{anything} = 0$$

$$0 \cdot \cos(2x) = 0$$

$\sin x = 0$ $\quad \cos(2x) = 0$

$x = 0, \pi, \cancel{2\pi}$ $\quad 2x = \pi/2, 3\pi/2, 5\pi/2, 7\pi/2$

$x = \pi/4, 3\pi/4, \cancel{5\pi/4, 7\pi/4}$



$V(t)$ [+ | − | +]
0 .785 2.356 π

$$\boxed{\begin{array}{l} \text{stopped: } t = .785, 2.356 \text{ sec. b/c } V(t)=0 \\ \text{right: } (0, .785)\ (2.356, 3.141) \text{ b/c } V(t)>0 \\ \text{left: } (.785, 2.356) \text{ b/c } V(t)<0 \end{array}}$$

b) accel at $t=2$

$V'(t) = a(t)$ $\quad$ ft/sec^2

$a(2) \approx 1.64832$ [nDeriv(Y_1, X, 2)]

c) ave accel $[0, \pi]$

$$\frac{V(\pi) - V(0)}{\pi - 0} \text{ ft/sec}^2 \quad [(Y_1(\pi) - Y_1(0))/(\pi - 0)]$$

d) displacement on $[0, \pi]$

$$\int_0^{\pi} \sin t \cdot \cos(2t)\, dt$$

$\boxed{\approx -.66\overline{6} \text{ ft.}}$

e) ave. Velocity on $[0, \pi]$

$$\frac{1}{\pi - 0} \int_0^{\pi} \sin t \cdot \cos(2t)\, dt$$

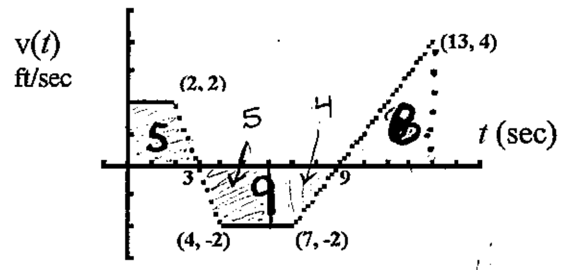
$$\approx \frac{1}{\pi}(-.66\overline{6}) \approx \boxed{-.2122 \text{ ft/sec}}$$

f) total distance on $[0, \pi]$

$$\int_0^{\pi} |v(t)|\, dt$$

$\boxed{\approx 1.2189 \text{ ft}}$

6) An object is moving along an x-axis and its velocity $v(t)$ is given in the graph below. The velocity is measured in feet/sec. The object's position is $x(0) = 3$



a) Find the position at each time.

t = 0 $x(0) = 3$

$\frac{1}{2}(2)(3+1)$

4

t = 9 $x(9) = 3 + \int_0^9 v(t)\,dt = 3 + 5 - 9 = \boxed{-1}$

t = 13 $x(13) = 3 + \int_0^{13} v(t)\,dt = 3 + 5 - 9 + 8 = \boxed{7}$

b) Find the displacement on $[0,13]$.

$\int_0^{13} v(t)\,dt = 5 - 9 + 8 = \boxed{4 \text{ ft}}$

c) Find the total distance traveled on $[0,13]$.

$\int_0^{13} |v(t)|\,dt = 5 + 9 + 8 = \boxed{22 \text{ ft}}$

d) Find the average velocity of the object on $[0,13]$.

$\frac{1}{13-0}\int_0^{13} v(t)\,dt = \frac{1}{13}(4) = \boxed{\frac{4}{13} \text{ ft/sec}}$ or $\frac{S(13) - S(0)}{13 - 0} = \frac{7-3}{13}$

e) Find when the object is at 3, other than at t = 0. Justify.

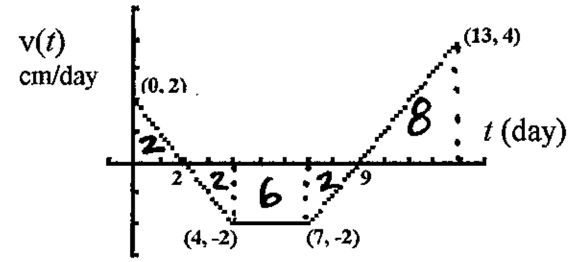
displacement $= 0 \therefore \int_0^m v(t)\,dt = 0$ $m = 6$ sec., and (between 11 and 12)

f) At the end of the trip (t = 13), is the object left, right, or at the same place from where it started?

$x(0) = 3$ and $x(13) = 7$ $\therefore$ the object is 4 ft. on the right side of the starting position

g) Find a(3) $= v'(3) = \boxed{-2 \text{ ft/sec}^2}$

7) A snail is moving along an x-axis and its velocity $v(t)$ is given in the graph below. The velocity is measured in cm/day.



Show all analyses that leads to your solutions.

a) Find the snail's acceleration at t = 2. $a(2) = v'(2) = -1 \text{ cm/day}^2$

b) Find the displacement of the snail on [0,4]

$$\int_0^4 v(t)\,dt = \boxed{0 \text{ cm.}}$$

c) Find the displacement of the snail on [4,11]

$$\int_4^{11} v(t)\,dt = -8 + \tfrac{1}{2}(2)(2) = -8 + 2 = \boxed{-6 \text{ cm}}$$

d) Find the total distance the snail traveled on [0,13]

$$\int_0^{13} |v(t)|\,dt = 2 + 10 + 8 = 20 \text{ cm}$$

have student's write

$$\boxed{P'(t) = v(t)}$$

e) Given that at $t = 4$ the snail was at position $x = 10$ on an x-axis. Find the position of the snail at each time

Help students set-up

t = 4 $P(4) = 10$

t = 9 $P(9) = P(4) + \int_4^9 v(t)\,dt = 10 - 8 = \boxed{2}$

t = 13 $P(13) = P(4) + \int_4^{13} v(t)\,dt = 10 - 8 + 8 = \boxed{10}$

t = 2 $P(2) = P(4) + \int_4^2 v(t)\,dt = P(4) - \int_2^4 v(t)\,dt = 10 - (-2) = \boxed{12}$

t = 0 $P(0) = P(4) + \int_4^0 v(t)\,dt = P(4) - \int_0^4 v(t)\,dt = 10 - [-2+2] = \boxed{10}$

f) Write a function P(t) to determine the position of the snail on the x-axis at any given time t.

~~P(x)~~ = $P(t) = P(4) + \int_4^t v(t)\,dt$

g) At time t = 9, is the snail left, right, or at the same place from the snail position at t = 4? Justify.

Left b/c $P(4) = 10$ and $P(9) = 2$ $\therefore$ the snail is 8 cm to the left.

h) Find *when* the object is at 10, other than at $t = 4$. Justify.

$t = 0$ and $t = 13$ b/c $\int_4^0 v(t)\,dt = 0$

$\int_4^{13} v(t)\,dt = 0$

8) The rate of milk consumption in the United States is given by $C(t)=35.08e^{t/30}$ where t is the number of years after Jan 1, 1980 and measured in kiloliters per year.

a) Find the rate of milk consumption in 1985.

$$C(5) = 35.08e^{5/30} \approx \boxed{41.442 \text{ kl/yr}}$$

Type 35.08e^(x/30) into y1
Then on the home screen type
y1(5) enter

b) Find the total amount of milk consumed from 1980-2000.

$$\int_0^{20} (35.08e^{t/30})\,dt \approx \boxed{997.395 \text{ kl}}$$

c) Find the total amount of milk consumed from 1988-1990.

$$\int_8^{10} (35.08e^{t/30})\,dt \approx \boxed{94.732 \text{ kl}}$$

d) Find the average rate of milk consumption from 1980 to 1990.

$$\frac{1}{10-0}\int_0^{10} (35.08e^{t/30})\,dt \approx \boxed{41.634 \text{ kl/yr}}$$

9) Below is a chart of velocities of a ball rolling down an incline in meters per minute.

	mins	m/min
	0	0
3	3	15
2	5	25
3	8	20
4	12	30

a) Find the average acceleration on $[0,12]$ min.

$$\frac{V(12)-V(0)}{12-0} = \frac{30-0}{12-0} = \frac{5}{2} \text{ m/min}^2$$

b) When is acceleration negative?

$(5,8)$ $\quad v'(t) < 0$

c) Estimate the acceleration of 3 minutes

$$\frac{V(5)-V(0)}{5-0} = \frac{25-0}{5-0} = 5 \text{ m/min}^2$$

d) Use a right rectangular approximation to estimate $\int_0^{12} v(t)dt \approx [3(\ )+2(\ )+3(\ )+4(\ )]$

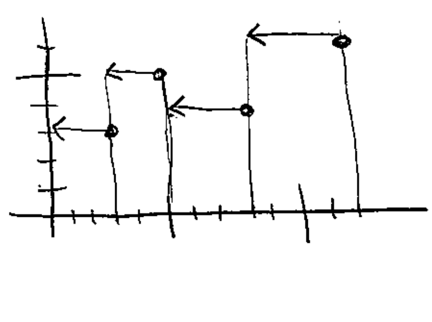
$\approx [3(15)+2(25)+3(20)+4(30)]$

≈ 275m.

e) Using correct units, what is *d)* telling you?

The ball rolled 275 meters in 12 mins.

f) Find the average velocity of the ball on $[0,12]$



$$\frac{1}{12-0}\int_0^{12} v(t)\,dt$$

$$\approx \frac{1}{12 \text{ min}}(275 \text{m})$$

$$\approx \frac{275}{12} \text{ m/min}$$

$$\boxed{\approx 22.917 \text{ m/min}}$$

For problems 10, 11, 12. (velocities measured in cm/hr.)

a) Determine when the object is moving right, left and stopped

b) Determine the *acceleration* of the object at t = 1.

c) Determine the *average acceleration* of the object over the given time interval *(set-up only)*

d) Determine the *displacement* of the object over the given time interval

e) Determine the *average velocity* of the object over the given time interval *(set-up only)*

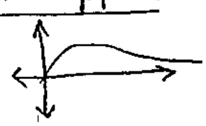
f) Determine the *total distance* traveled by the object over the given time interval

10) *(calc. ok)* $v(t) = 6\sin 3t$ $\qquad 0 \le t \le \frac{\pi}{2}$

11) *(calc. ok)* $v(t) = \frac{t}{1+t^2}$ $\qquad 0 \le t \le 3$

a) right, left, stopped

$\frac{t}{1+t^2} \stackrel{?}{=} 0$



stopped: $t = 0$ hr.

right: $(0, 3]$ $v > 0$

left: never $v < 0$

b) accel @ $t = 1$

$a(1) = v'(1) = 0$ cm/hr²

c) ave. accel $[0, 3]$ $\frac{v(3) - v(0)}{3-0} = .1$ cm/hr²

d) Displacement $[0, 3]$

$\int_0^3 v(t)\,dt \approx 1.151$ cm

e) Ave. Velocity $[0, 3]$

$\frac{1}{3-0}\int_0^3 v(t)\,dt \approx .3837$ cm/hr.

f) total distance $[0, 3]$

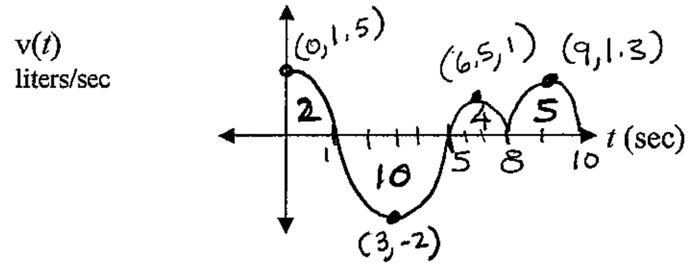
$\int_0^3 |v(t)|\,dt \approx 1.151$ cm

why is the displacement = total distance

b/c the velocity is positive on the interval

12) *(no calc.)* $v(t) = 6t^2 + 18t - 12$ $\qquad 0 \le t \le 3$

13) Water is poured into a container and drained out. The velocity v(t) of the water is given in the graph below.



Show all analyses that leads to your solutions.

a) What is the velocity of the water at t = 3? $V(3) = \boxed{-2 \text{ liters/sec}}$

b) Is the speed of the water going into the container greatest at t = 3 or t = 9? Justify.

Speed at $t=3$ $|V(3)| = 2$ liters/sec
Speed at $t=9$ $|V(9)| = 1.3$ liters/sec
∴ $\boxed{\text{Speed is greatest at } t = 3 \text{ seconds}}$

c) Find the displacement of the water on $[1,8]$

$\int_1^8 v(t)\,dt = -10 + 4 = \boxed{-6 \text{ liters}}$

d) Find the displacement of the water on $[5,10]$

$\int_5^{10} v(t)\,dt = 4 + 5 = \boxed{9 \text{ liters}}$

e) Find the total amount of water poured into and drained from the container on $[0,10]$

$\int_0^{10} |v(t)|\,dt = 2 + 10 + 4 + 5 = \boxed{21 \text{ liters}}$

f) Given that container held **3** liters of water at $t = 5$.
Find the amount of water in the container at each time

t = 8 $L(5) + \int_5^8 v(t)\,dt = 3 + 4 = \boxed{6 \text{ liters}}$

t = 10 $L(5) + \int_5^{10} v(t)\,dt = 3 + 4 + 5 = \boxed{12 \text{ liters}}$

t = 1 $L(5) + \int_5^1 v(t)\,dt = 3 - \int_1^5 v(t)\,dt = 3 - (-10) = \boxed{13 \text{ liters}}$

t = 0 $L(5) + \int_5^0 v(t)\,dt = 3 - \int_0^5 v(t)\,dt = 3 - (-10 + 2) = \boxed{11 \text{ liters}}$

g) Write a function L(t) to determine the amount of water in the container at any given time t.

L(t) = $L(5) + \int_5^t v(t)\,dt$

h) At what time does the container have the most liters of water on the 10 minute interval? Justify.

possible places are when $v(t)$ changes from + to − or an end pt.
$t = 1$ or $t = 10$ $L(1) = 13$ and $L(10) = 12$ liters ∴ the container holds the most water at $\boxed{t = 1 \text{ seconds}}$

i) Within the 10 minute time interval, did the container hold 14 liters of water? Justify.

The greatest amount of water in the container on the interval is 13 liters. ∴ the container never hold 14 liters.

14) The rate of consumption of fuel in Arkansas after 1990 (in billions of barrels per year) is modeled by the function $C = 25.01e^{t/15}$.

a) Find the rate of fuel consumption in 1992.

$C(2) = 25.01e^{2/15} \approx 28.577$ bb/yr.

b) Find the total amount consumed from January 1, 1990 to January 1, 1995.

$\int_0^5 25.01e^{t/15}\,dt \approx 148.414$ bb

c) Find the total amount consumed from January 1, 1993 to January 1, 1998.

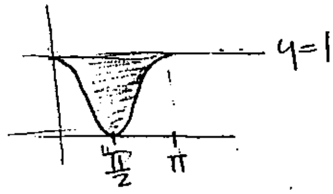
$\int_3^8 25.01e^{t/15}\,dt \approx 181.273$ bb

d) Find the average rate of consumption of fuel in Arkansas from January 1, 1993 to January 1, 1998.

$\frac{1}{8-3}\int_3^8 25.01e^{t/15}\,dt \approx 36.254$ bb/yr.

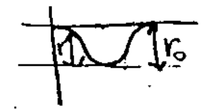
15 ~~18~~

$y=1$
$y=\cos^2 x$
$x=0$
$x=\pi$



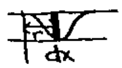
a) Find the area enclosed $A=\int_0^{\pi}(1-\cos^2 x)\,dx \approx \boxed{1.571}$

b) Find the volume generated by revolving the region about x-axis. USE WASHERS



$V=\pi\int_0^{\pi}\left[(1-0)^2-(\cos^2 x-0)^2\right]dx \approx \boxed{6.1685}$

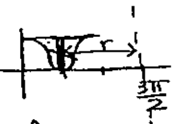
c) About y-axis Use Shells



$V=2\pi\int_0^{\pi}(x-0)(1-\cos^2 x)\,dx$

d) About $x=\dfrac{3\pi}{2}$ *(set-up, but do not evaluate)*

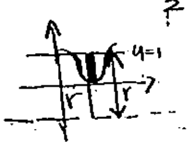
Use shells



$V=2\pi\int_0^{\pi}\left(\frac{3\pi}{2}-x\right)(1-\cos^2 x)\,d$

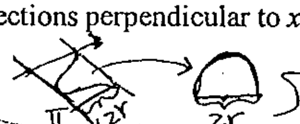
e) About $y=-1$ *(set-up, but do not evaluate)*

use washers



$V=\pi\int_0^{\pi}(1-(-1))^2-(\cos^2 x-(-1))^2\,d$

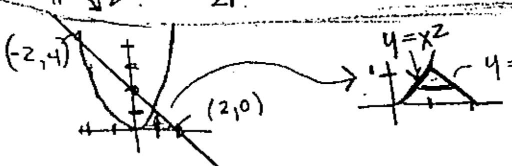
f) Cross sections perpendicular to x-axis are semi-circles *(set-up, but do not evaluate)*



$r=\frac{1}{2}(1-\cos^2 x) \Rightarrow V=\pi\int_0^{\pi}\left(\frac{1-\cos^2 x}{2}\right)dx$

16 ~~19~~

$y=x^2$
$x+y=2$
$y=0$



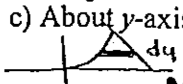
$y=2-x \Rightarrow x=2-y$
$y=x^2 \Rightarrow x=\pm\sqrt{y}$

a) Find the area enclosed USE dy. $A=\int_0^1\left((2-y)-\sqrt{y}\right)dy \approx \boxed{.833}$

b) Find the volume generated by revolving the region about x-axis

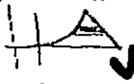
Shells $V=2\pi\int_0^1(y-0)(2-y-\sqrt{y})\,dy \approx \boxed{1.6755}$

c) About y-axis ~ WASHER

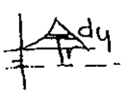


$V=\pi\int_0^1(2-y-0)^2-(\sqrt{y}-0)^2\,dy \approx \boxed{5.7595}$

d) About $x=-1$ *(set-up, but do not evaluate)* WASHERS



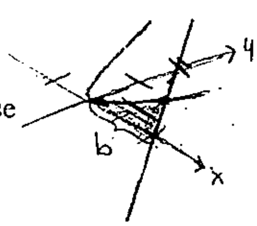
$V=\pi\int_0^1(2-y-(-1))^2-(\sqrt{y}-(-1))^2\,dy$



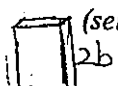
e) About $y=-2$ *(set-up, but do not evaluate)*

$V=2\pi\int_0^1(y-(-2))(2-y-\sqrt{y})\,dy$

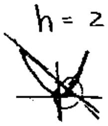
f) Cross sections perpendicular to y-axis are rectangles with height twice the base *(set-up, but do not evaluate)*



$A=bh$
$h=2b$



$V=\int_0^b(2-y-\sqrt{y})(2(2-y-\sqrt{y}))\,dy$



g) Find the value k that divides the area *(from part 'a')* into 2 equal parts. *(set-up, but do not evaluate)*

$\int_0^k(2-y-\sqrt{y})\,dy=\int_k^1(2-y-\sqrt{y})\,dy$