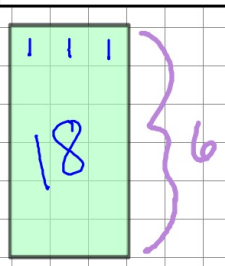
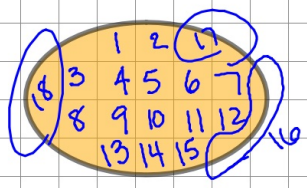
Section 6.1: Evaluating Definite Integrals using Estimation

determine the area of the rectangle

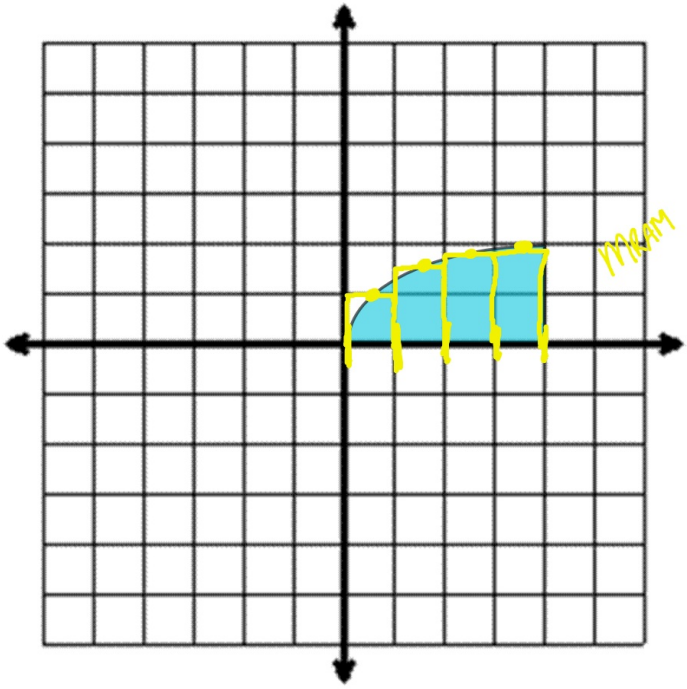


$= 18 \text{ units}^2$



determine the area of the shaded region $\approx 18 \text{ units}^2$

determine the area of the shaded region



RAM
Rectangular Approximation Method

Riemann Sum

RRAM (right)

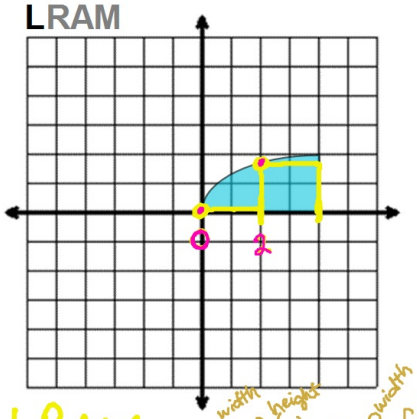
LRAM (left)

MRAM (middle)

Determine the *area* trapped between a curve and the x-axis

1. a) $f(x) = \sqrt{x}$ [0, 4] n = 2

LRAM



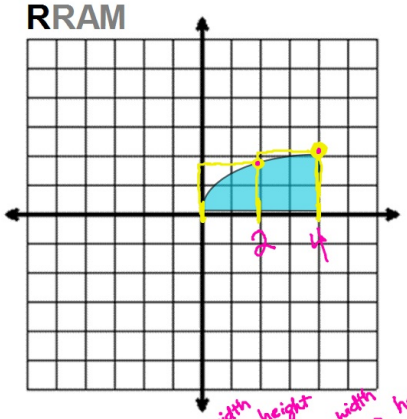
$LRAM \approx \underset{\text{width}}{2} \cdot \underset{\text{height}}{f(0)} + \underset{\text{width}}{2} \cdot \underset{\text{height}}{f(2)}$

$\approx 2 \cdot 0 + 2 \cdot \sqrt{2}$

$\approx 2\sqrt{2}$

$\approx \underset{\text{width}}{2}\left(\underset{\text{height}}{\sqrt{0}} + \underset{\text{height}}{\sqrt{2}}\right)$

RRAM



$RRAM \approx \underset{\text{width}}{2} \cdot \underset{\text{height}}{f(2)} + \underset{\text{width}}{2} \cdot \underset{\text{height}}{f(4)}$

$\approx 2 \cdot \sqrt{2} + 2 \cdot \sqrt{4}$

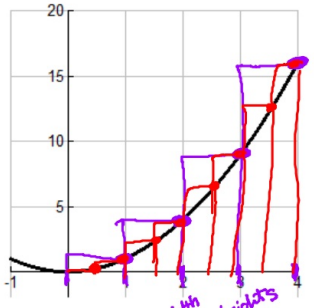
$\approx 2\sqrt{2} + 4$

$\approx \underset{\text{width}}{2}\left(\underset{\text{height}}{\sqrt{2}} + \underset{\text{height}}{\sqrt{4}}\right)$

Determine the *area* trapped between a curve and the x-axis

2. a) $f(x) = x^2$ [0, 4] n = 4

RRAM



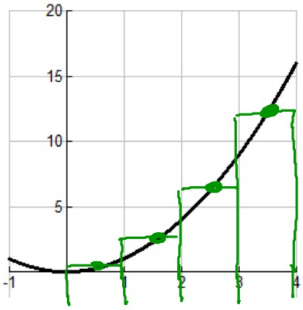
$RRAM \approx \underset{\text{width}}{1}\left[\underset{\text{heights}}{f(1) + f(2) + f(3) + f(4)}\right]$

$\approx 1\left[(1)^2 + (2)^2 + (3)^2 + (4)^2\right]$

$\approx 1 \cdot 1 + 4 + 9 + 16$

≈ 30

MRAM



$MRAM \approx 1\left[f\left(\frac{1}{2}\right) + f\left(\frac{3}{2}\right) + f\left(\frac{5}{2}\right) + f\left(\frac{7}{2}\right)\right]$

$\approx 1\left[\left(\frac{1}{2}\right)^2 + \left(\frac{3}{2}\right)^2 + \left(\frac{5}{2}\right)^2 + \left(\frac{7}{2}\right)^2\right]$

≈ 21

n=8 RRAM

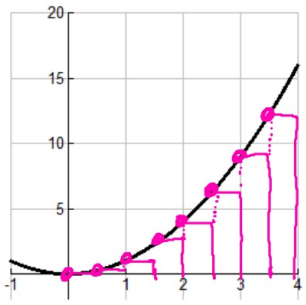
$RRAM \approx \frac{1}{2}\left((\frac{1}{2})^2 + (1)^2 + (\frac{3}{2})^2 + (2)^2 + (\frac{5}{2})^2 + (3)^2 + (\frac{7}{2})^2 + 4^2\right)$

$\approx 25\frac{1}{2} \approx 25.5$

Determine the *area* trapped between a curve and the x-axis

3. b) $f(x) = x^2$ **[0, 4]** **n = 8**

LRAM



$$LRAM \approx \frac{1}{2}\left(f(0)+f\left(\frac{1}{2}\right)+f(1)+f\left(\frac{3}{2}\right)+f(2)+f\left(\frac{5}{2}\right)+f(3)+f\left(\frac{7}{2}\right)\right)$$

$$\approx \frac{1}{2}\left(0+\frac{1}{4}+1+\frac{9}{4}+4+\frac{25}{4}+9+\frac{49}{4}\right)$$

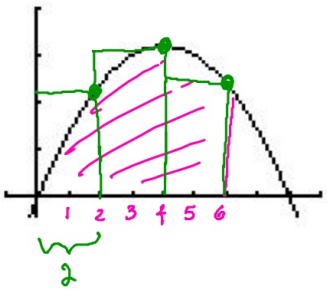
$$\approx \frac{1}{2}(35)$$

$$\approx 17.5$$

Determine the *area* trapped between a curve and the x-axis

4. a) $f(x) = 8x - x^2$ **[0, 6]**

n = 3 **RRAM**



$$RRAM \approx 2\left[f(2)+f(4)+f(6)\right]$$

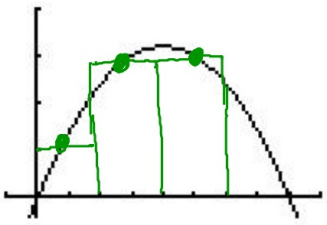
$$\approx 2\left[12+16+12\right]$$

$$\approx 2(40)$$

$$\approx 80 \text{ units}^2$$

b) $f(x) = 8x - x^2$ **[0, 6]**

n = 3 **MRAM**



$$MRAM \approx 2\left[f(1)+f(3)+f(5)\right]$$

$$\approx 2\left[7+15+15\right]$$

$$\approx 2(37)$$

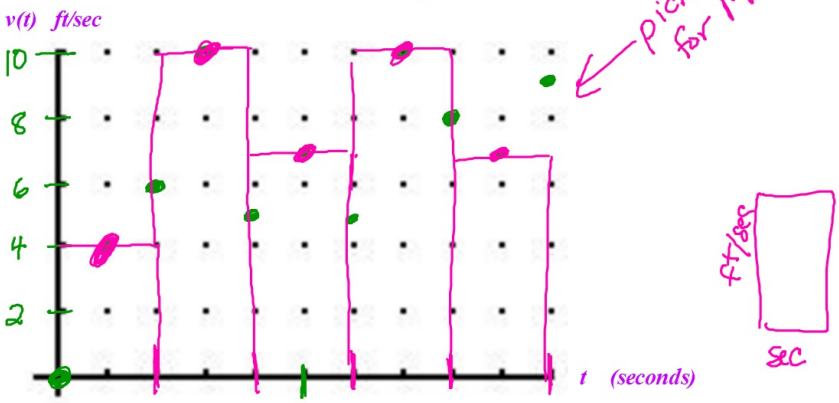
$$\approx 74$$

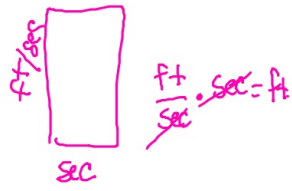
4. Application exercise

A remote control car's velocity was recorded.

Estimate the total distance driven within the given time frame.

t	v(t) ft/sec
0	0
1	4
2	6
3	10
4	5
5	7
6	5
7	10
8	8
9	7
10	9





a) Use a RRAM approximation with $n = 10$

$RRAM \approx 1(4+6+10+5+7+5+10+8+7+9)$

≈ 71 ft.

The remote control car travelled approx. 71 feet from [0,10] seconds

b) Use a MRAM approximation with $n = 5$

$MRAM \approx 2(4+10+7+10+7)$

$\approx 2(38)$

≈ 76 ft.

The remote control car travelled approx. 76 feet from [0,10] seconds