Chapter 5 (part II) Review

Key

Determine if the Mean Value Theorem applies to the following problems

1) $f(x) = x^2 + 5x$ on $[2, 4]$ yes

2) $f(x) = \dfrac{x^2 - x}{x}$ on $[-1, 2]$ no (not continuous)

3) $f(x) = \dfrac{x^2 - x}{x}$ on $[4, 6]$ yes

4) $f(x) = \ln(x-3)$ on $[3, 5]$ no
(3 is not in the domain of h(x))

Find the value of "x" that satisfies the M.V.T.

5) $f(x) = x^2 - 2x + 3$ on $[1, 3]$

$$\frac{f(3)-f(1)}{3-1} = f'(c)$$

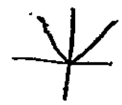
$$\frac{(9-6+3)-(1-2+3)}{2} = 2c-2$$

$$2 = 2c - 2$$

$$4 = 2c$$

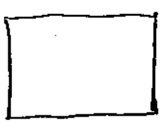
$$\boxed{c = 2}$$

6) $f(x) = |x|$ on $[-3, 3]$



no value

7) A gardener has 600 feet of fencing to enclose an area of land to develop. What dimensions should be used so that the gardener has the maximum area of land.



$P = 2l + 2w$

$600 = 2l + 2w$

$600 - 2l \div 2w$

$300 - l = w$

$A = l \cdot w$

$A = l(300 - l)$

$A = 300l - l^2$

$A' = 300 - 2l$

$A' = 0$

$300 - 2l = 0$

$300 = 2l$

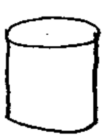
$l = 150$

$l = 150$ ft

$w = 150$ ft

The dimensions of the garden should be 150 ft × 150 ft.

8) As a design engineer you have been contracted to design a cylindrical culinary water supply tank to hold 230,000 cubic centimeters of water as inexpensively as possible. What dimensions will minimize the surface area of the tank?



given

$V = \pi r^2 h$

$\dfrac{230{,}000}{\pi r^2} = h$

$h = \dfrac{230000}{\pi\left(\frac{115{,}000}{\pi}\right)^{2/3}}$ cm

optimize

$SA = 2\pi r^2 + 2\pi r h$

$SA = 2\pi r^2 + 2\pi r\left(\dfrac{230{,}000}{\pi r^2}\right)$

$SA = 2\pi r^2 + \dfrac{460{,}000}{r}$

$SA' = 4\pi r - \dfrac{460{,}000}{r^2}$

$SA' = 0$

$4\pi r - \dfrac{460{,}000}{r^2} = 0$

$4\pi r = \dfrac{460{,}000}{r^2}$

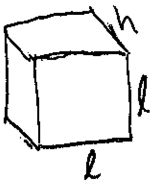
$4\pi r^3 = 460{,}000$

$r^3 = \dfrac{115{,}000}{\pi}$

$r = \sqrt[3]{\dfrac{115{,}000}{\pi}}$ cm

The radius should be $\sqrt[3]{\dfrac{115{,}000}{\pi}}$ cm and the height should be $\dfrac{230{,}000}{\pi\left(\sqrt[3]{\frac{115{,}000}{\pi}}\right)^2}$ cm.

9) A 500 ft^3 square-based, rectangular box is to be constructed to weigh as little as possible. What dimensions will minimize the surface area of the box?



have

$V = l^2 h$

$500 = l^2 h$

$\frac{500}{l^2} = h$

$h = \frac{500}{(\sqrt[3]{500})^2}$ ft

optimize

$SA = 2l^2 + 4lh$

$SA = 2l^2 + 4l\left(\frac{500}{l^2}\right)$

$SA = 2l^2 + \frac{2000}{l}$

$SA' = 4l - \frac{2000}{l^2}$

$SA' = 0$

$4l - \frac{2000}{l^2} = 0$

$4l = \frac{2000}{l^2}$

$4l^3 = 2000$

$l^3 = 500$

$l = \sqrt[3]{500}$ ft

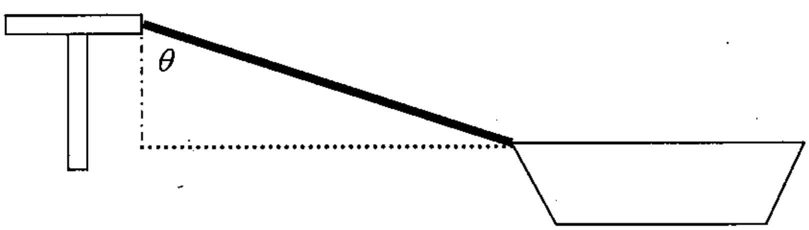
*Note: $\frac{500}{\sqrt[3]{500}^2} = 500^{1-\frac{2}{3}}$

$= \sqrt[3]{500}$

The length of the sides of the square is $\sqrt[3]{500}$ ft, and the hieght of the box is $\frac{500}{\sqrt[3]{500}^2}$* ft.

10) A boat pulled toward a dock by a rope. The dock is 6 feet above the top of the boat. The rope is hauled in at 2 feet per second.

a) How fast is the boat approaching the dock when 10 feet of rope is out?



Know

$\frac{dz}{dt} = -2$ ft/sec

want

$\frac{dx}{dt} = -2.5$ ft/sec

$x^2 + y^2 = z^2$

$x^2 + 36 = z^2$

$\cancel{2}x\frac{dx}{dt} = \cancel{2}z\frac{dz}{dt}$

$8\frac{dx}{dt} = 10(-2)$

$\frac{dx}{dt} = \frac{-20}{8} = -\frac{5}{2}$

The boat is approaching the dock at 2.5 ft/sec

b) At what rate is θ changing?

$\sin\theta = \frac{8}{10} = \frac{4}{5}$

Know

$\cos\theta = \frac{6}{z}$

$-\sin\theta\frac{d\theta}{dt} = \frac{-6}{z^2}\frac{dz}{dt}$

$-\frac{4}{5}\frac{d\theta}{dt} = \frac{-6}{100} \cdot -2$

$\frac{d\theta}{dt} = \frac{\cancel{12}^{3}}{\cancel{100}_{20}} \cdot \frac{-\cancel{5}^{1}}{\cancel{4}_{1}} = \frac{-3}{20}$

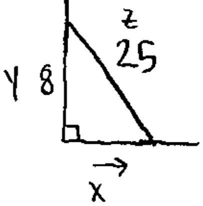
want

$\frac{d\theta}{dt} = \frac{-3}{20}$ rad/sec

The angle is changing at a rate of $-\frac{3}{20}$ rad/sec.

11) A ladder 25 ft. long leans against a vertical building. Answer the following questions given the bottom of the ladder sides away from the building horizontally at the rate of 1 ft./sec. and the top of the ladder is 8ft. from the ground.

a) At that instant, what is the *speed* the ladder siding down the building?



given

$\frac{dx}{dt} = 1 \text{ ft/sec}$

when $y = 8$

want

$\frac{dy}{dt} = -\frac{\sqrt{561}}{8}$ ft/sec

$x^2 + 8^2 = 25^2$

$x^2 = 625 - 64$

$x^2 = 561$

$x = \sqrt{561}$

$x^2 + y^2 = 25^2$

$2x\frac{dx}{dt} + 2y\frac{dy}{dt} = 0$

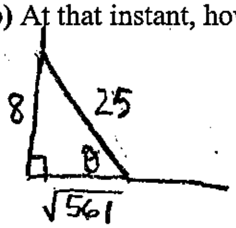
$2x\frac{dx}{dt} = -2y\frac{dy}{dt}$

$2\sqrt{561} \cdot 1 = -2(8)\frac{dy}{dt}$

$\frac{dy}{dt} = \frac{2\sqrt{561}}{-16}$

The ladder is sliding down the building at $\frac{\sqrt{561}}{8}$ ft/sec.

b) At that instant, how fast is the angle between the ladder and the ground changing?



$\sin\theta = \frac{y}{25}$

$\cos\theta \frac{d\theta}{dt} = \frac{1}{25}\frac{dy}{dt}$

$\frac{\sqrt{561}}{25}\frac{d\theta}{dt} = \frac{1}{25}\left(-\frac{\sqrt{561}}{8}\right)$

$\frac{d\theta}{dt} = \frac{-\sqrt{561}}{200} \cdot \frac{25}{\sqrt{561}}$

want

$\frac{d\theta}{dt} = -\frac{1}{8}$ rad/sec

The angle between the ladder and the ground is decreasing at 1/8 rad/sec

12) Sand is falling onto a conical pile at the rate of 10 cubic feet per minute. The diameter of the base of the cone is three times the height. At what rate is the height of the pile changing when it is 15 ft. high?

given

$\frac{dV}{dt} = 10 \text{ ft}^3/\text{min}$

want

$\frac{dh}{dt} = \frac{8}{405\pi}$ ft/min

when $h = 15$

$d = 3h$

$2r = 3h$

$r = \frac{3}{2}h$

$V = \frac{1}{3}\pi r^2 h$

$V = \frac{1}{3}\pi\left(\frac{3}{2}h\right)^2 h$

$V = \frac{1}{3}\pi \cdot \frac{9}{4}h^2 \cdot h$

$V = \frac{3}{4}\pi h^3$

$V' = \frac{9}{4}\pi h^2 \frac{dh}{dt}$

$10 = \frac{9}{4}\pi(15)^2\frac{dh}{dt}$

$\frac{4}{9} \cdot \frac{\cancel{10}^{\,2}}{\cancel{225}_{\,45}\pi} = \frac{dh}{dt}$

$\frac{dh}{dt} = \frac{8}{405\pi}$

The height of the pile is changing at $8/405\pi$ ft/min

13) A rocket is shot up from the ground. The position function of the rocket is $s(t) = -16t^2 + 40t$, where t is measured in seconds and $s(t)$ is in feet.

$s' = v$

$s'' = v' = a$

a) Find the position of the rocket after 2 seconds.

$$s(2) = -16(2)^2 + 40(2)$$
$$= -64 + 80$$
$$= 16$$

The rocket is up 16 ft.

b) Find the instantaneous velocity of the rocket at $t = 2$.

$$s' = v = -32t + 40$$

$$v(2) = -32(2) + 40$$
$$= -64 + 40$$
$$= -24$$

The velocity is -24 ft/sec.

c) Find the average velocity of the rocket for the first 2 seconds.

$$\frac{s(2) - s(0)}{2 - 0} = \frac{16 - 0}{2} = 8 \text{ ft/sec}$$

The average velocity is 8 ft/sec

d) When did the rocket hit the ground?

$$s(t) = 0$$

$$-16t^2 + 40t = 0$$

$$-8t(2t - 5) = 0$$

$$t = 0,\ 5/2$$

The rocket hits the ground at 5/2 seconds

e) Find the speed of the rocket when it hits the ground.

$$\left|v\left(\tfrac{5}{2}\right)\right| = \left|-32\left(\tfrac{5}{2}\right) + 40\right|$$
$$= |-80 + 40|$$
$$= 40$$

The speed of the rocket is 40 ft/sec when it hits the ground.