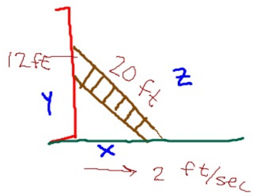
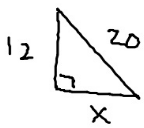
Section 5.6 Related Rates

1) A ladder 20ft long leans against the wall of a vertical building.
If the bottom of the ladder slides away from the building horizontally at a rate of 2 ft/sec, how *fast* is the *ladder sliding down* the building when the top of the ladder is 12ft above the ground?

Drawing





$$x^2 + 12^2 = 20^2$$
$$x^2 = 400 - 144$$
$$x = \sqrt{256}$$
$$x = 16$$

Know

$\frac{dx}{dt} = 2$ ft/sec

when $y = 12$

$$x^2 + y^2 = z^2$$
$$x^2 + y^2 = 400$$
$$2x \cdot \frac{dx}{dt} + 2y\frac{dy}{dt} = 0$$
$$2 \cdot 16 \cdot 2 + 2 \cdot 12 \frac{dy}{dt} = 0$$
$$64 + 24\frac{dy}{dt} = 0$$
$$24\frac{dy}{dt} = -64$$

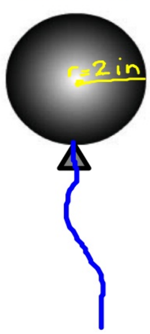
Want

$\frac{dy}{dt} = -\frac{8}{3}$ ft/sec

The ladder is sliding down the building at the rate of $\frac{8}{3}$ ft/sec. when the ladder is 12ft above the ground.

2) Air is being pumped into a spherical balloon at the rate of 4.5 cubic inches per minute.
Find the *rate of change* of the *radius* when the radius is 2 inches.

Drawing



Know

$\frac{dV}{dt} = 4.5$ in³/min

when $r = 2$in

$$V = \frac{4}{3}\pi r^3$$
$$\frac{dV}{dt} = 4\pi r^2 \frac{dr}{dt}$$
$$4.5 = 4\pi(2)^2 \cdot \frac{dr}{dt}$$
$$\frac{dr}{dt} = \frac{4.5}{16\pi}$$

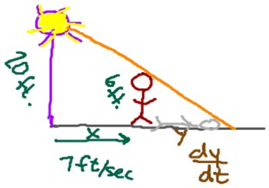
Want

$\frac{dr}{dt} = \frac{4.5}{16\pi}$ in/min

The radius is changing at the rate of $\frac{4.5}{16\pi}$ in/min when the radius is 2in.

3) A person 6ft tall is walking away from a streetlight 20ft high at the rate of 7 ft/sec. At what *rate* is the *length* of the person's shadow increasing?

Drawing	Know	Want



Know

$\frac{dx}{dt} = 7$ ft/sec

$\frac{20}{x+y} = \frac{6}{y}$

$20y = 6(x+y)$

$20y = 6x + 6y$

$14y = 6x$

$14\frac{dy}{dt} = 6\frac{dx}{dt}$

$14\frac{dy}{dt} = 6 \cdot 7$

$\frac{dy}{dt} = \frac{42}{14} = 3$

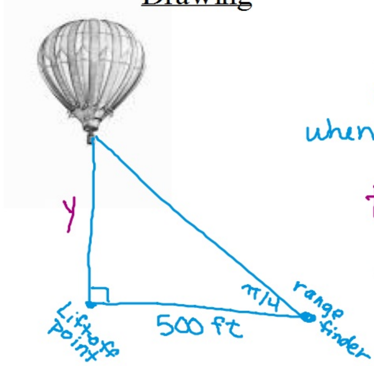
Want

$\frac{dy}{dt} = 3$ ft/sec

The shadow is increasing at a rate of 3 ft/sec.

4) A hot-air balloon rising straight up from a level field is tracked by a range finder 500ft. from the lift-off point. At the moment the range finder's elevation angle is $\frac{\pi}{4}$, the *angle is increasing* at the rate of 0.14rad/min. How *fast* is the *balloon rising*?

Drawing	Know	Want



Know

$\frac{d\theta}{dt} = 0.14$ rad/min

when $\theta = \frac{\pi}{4}$

$\tan\theta = \frac{y}{500}$ $\frac{1}{500} \cdot y$

$\sec^2\theta \cdot \frac{d\theta}{dt} = \frac{1}{500}\frac{dy}{dt}$

$\underbrace{\sec^2\frac{\pi}{4}}_{(\sqrt{2})^2} \cdot (0.14) = \frac{1}{500} \cdot \frac{dy}{dt}$

$500 \cdot 2(0.14) = \frac{dy}{dt}$

$140 = \frac{dy}{dt}$

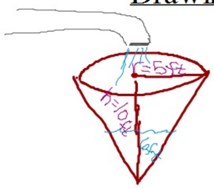
Want

$\frac{dy}{dt} = 140$ ft/min

The balloon is rising at a rate of 140 ft/min when $\theta = \frac{\pi}{4}$.

5) Water runs into a conical tank at the rate of 9 cubic ft/min.
The tank stands point down and has a height of 10ft and a base radius of 5ft,
how fast is the *water level rising* when the water is 6ft deep.

Drawing | Know | Want



$\frac{r}{h} = \frac{5}{10}$

$r = \frac{1}{2}h$

$\frac{dV}{dt} = 9$ ft³/min when $h = 6$ ft

$V = \frac{1}{3}\pi r^2 h$

$V = \frac{1}{3}\pi\left(\frac{1}{2}h\right)^2 h$

$V = \frac{1}{3}\pi \cdot \frac{1}{4}h^2 \cdot h$

$V = \frac{1}{12}\pi h^3$

$\frac{dV}{dt} = \frac{3}{12}\pi h^2 \frac{dh}{dt}$

$9 = \frac{\pi}{4} \cdot 6^2 \cdot \frac{dh}{dt}$

$9 = 9\pi \frac{dh}{dt}$

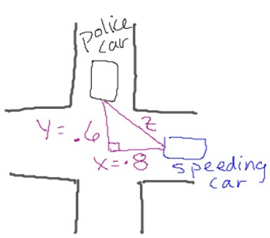
$\frac{1}{\pi} = \frac{dh}{dt}$

$\frac{dh}{dt} = \frac{1}{\pi}$ ft/min.

The water level is rising at a rate of $\frac{1}{\pi}$ ft/min. when the height of the water is 6 ft.

for fun . . .

6) A police cruiser, approaching a right–angle intersection from the north, is chasing a speeding car that has turned the corner and is now moving straight east. When the cruiser is 0.6 miles north of the intersection and the car is 0.8 miles to the east, the police determine with radar that the distance between them and the car is increasing at 20 mph. If the police cruiser is moving at 60 mph at the instant of measurement, what is the *speed of the car*?



$.6^2 + .8^2 = z^2$

$z = 1$

Know

$\frac{dy}{dt} = -60$ mph

$\frac{dz}{dt} = 20$ mph

$x^2 + y^2 = z^2$

$2x\frac{dx}{dt} + 2y\frac{dy}{dt} = 2z\frac{dz}{dt}$

$2(.8)\frac{dx}{dt} + 2(.6)(-60) = 2(1)(20)$

$1.6\frac{dx}{dt} = 40 + 72$

$\frac{dx}{dt} = \frac{112}{1.6} = 70$

Want

$\frac{dx}{dt} = 70$ mph

The car is going 70 m.p.h.