Sec 6.4: Exponential Growth and Decay Notes

1) Jack and Jill have $ to deposit in the bank.

The bank offers a 10% rate of interest

	Jack — \$100	Jill — \$1000
1st year:	\$110	\$1,100
2nd year:	\$121	\$1,210

\$ invested = y_0

rate \$ are increasing = $\frac{dy}{dt}$

In this case

$$\frac{dy}{dt} = .10y$$

rate

a) I want to know how much \$ they will have in 20 years.

General Format

$$\frac{dy}{dt} = ky$$

$$\int \frac{1}{y}\,dy = \int k\,dt$$

$$\ln|y| = kt + C$$

$$e^{kt+C} = y$$

$$\left.\begin{array}{l} y = Ce^{kt} \\ P = P_0 e^{rt} \\ y = y_0 e^{rt} \end{array}\right\} \text{All of the general formats}$$

General form for finding \$ in 20 year.

$$y = y_0 e^{.10t}$$

y_0 = initial investment

y = end amount

t = time

Jack

$$y = 100e^{.10(20)}$$

$$y \approx \$738.91$$

b) When will their money double?

time

$$\frac{2 \cdot 100}{100} = \frac{100e^{.10t}}{100}$$

$$2 = e^{.10t}$$

$$\ln 2 = \ln e^{.10t}$$

$$\ln 2 = .10t$$

$$t = \frac{\ln 2}{.10} \approx 6.9314 \text{ yrs}$$

$$\frac{2 \cdot 1000}{1000} = \frac{1000e^{.10t}}{1000}$$

$$2 = e^{.10t}$$

2) The bacteria in a certain culture increases continually at a rate directly proportional to the number present.

a) Write a differential equation to represent the problem.

$$\frac{dy}{dt} = ky$$

b) If there is initially 400 bacteria. Find P(t) in terms of $\boldsymbol{k}$ and $\boldsymbol{t}$.

P(t) = population @ any time "t"

$$\left(\frac{dt}{y}\right)\frac{dy}{dt} = ky\left(\frac{dt}{y}\right)$$

$$\int \frac{1}{y}\,dy = \int k\,dt$$

$$\ln|y| = kt + C$$

$$e^{kt+C} = y$$

$$y = Ce^{kt}$$

Work required on AP Test

$$y = Ce^{kt}$$

$$P(t) = Ce^{kt}$$

$$P(0) = 400 = Ce^{k(0)}$$

$$400 = C$$

$$P(0) = 400$$

$$P(t) = 400e^{kt}$$

c) If the number of bacteria triples in 6 hrs. – how many bacteria are there in 12 hrs.?

$$3(400) = 400e^{k(6)}$$

$$3 = e^{6k}$$

$$\ln(3) = \ln(e^{6k})$$

$$\ln(3) = 6k$$

$$k = \frac{\ln(3)}{6}$$

store in calculator

$$? = 400e^{k(12)}$$

$$? = 400e^{\frac{\ln 3}{6}\cdot 12}$$

$$P(12) = 3600 \text{ bacteria}$$

3) In ~~1992~~ 1998 I was interested in finding out if there was life on Mars. I found out there was actually 18 Martians living on Mars. I heard the other day there are now (~~2007~~ 2013) 59 Martians.

a) What is their rate of growth?

$$59 = 18e^{k(15)}$$

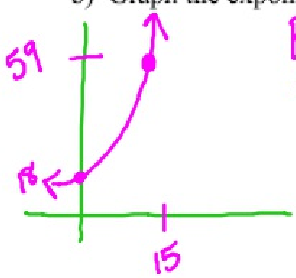
$$\frac{59}{18} = e^{15k}$$

$$\ln\left(\frac{59}{18}\right) = 15k$$

$$k = \frac{\ln\left(\frac{59}{18}\right)}{15}$$

rate of growth ≈ 0.079144

b) Graph the exponential function that represents their rate of growth.



$P(0) = 18$

$P(15) = 59$

What does $P(10)$ represent?

Is it a reasonable question?

4) You've been hired as a scientist to do carbon dating.

Carbon-14 dating uses 5700 yrs. for it's ½ life

(half life: population has decreased by ½)

a) Find the age of a sample in which 10% of the radioactive nuclei originally present have decayed.

(note: 90% remains)

<u>1st: Find the rate</u>

$$\frac{1}{2}P_0 = P_0 e^{5700k}$$

$$\frac{1}{2} = e^{5700k}$$

$$\ln\left(\frac{1}{2}\right) = 5700k$$

$$k = \frac{\ln\left(\frac{1}{2}\right)}{5700}$$

<u>2nd: Find the age when 90% remains</u>

$$90\% P_0 = P_0 e^{kt}$$

$$.9 = e^{kt}$$

$$\ln(.90) = kt$$

$$\frac{\ln(.90)}{k} = t$$

$$t \approx 866.418 \text{ yrs.}$$

memorize

Newton's Law of Cooling
(The temperature of a liquid and the surrounding temperature have an effect on how fast the liquid will cool)

Formula $T = (T_0 - T_s)e^{kt} + T_s$

T = end temperature
T_0 = starting temperature
T_S = surrounding temperature
k = rate
t = time

(Note: Cooling is an exponential function. We know that when a liquid cools, the lower the temperature of the liquid the faster it loses heat.)

You have hot cocoa when you go snowboarding / sledding etc.
It's 20° outside. When you test the temperature of the cocoa, it is 130°, then 10 minutes later it's 120°

a) How long will it take to get to 95° ?

1st: Find the *rate* the cocoa is cooling

$$120 = (130-20)e^{k(10)} + 20$$
$$100 = (110)e^{10k}$$
$$\frac{10}{11} = e^{10k}$$
$$\ln\left(\frac{10}{11}\right) = 10k$$
$$k = \frac{\ln\left(\frac{10}{11}\right)}{10}$$
$$k \approx -0.0095$$

2nd: How long to get to 95° ?

$$95 = (130-20)e^{kt} + 20$$
$$75 = 110e^{kt}$$
$$\frac{75}{110} = e^{kt}$$
$$\ln\left(\frac{75}{110}\right) = kt$$
$$t = \frac{\ln\left(\frac{75}{110}\right)}{k}$$
$$t \approx 40.18378 \text{ mins.}$$

b) Find the temperature of the cocoa in 1 hour.

$$T = (130^\circ - 20)e^{60k} + 20$$
$$T \approx 82.092^\circ F$$