Sec 7.2: Integration by Substitution

Review:

1st: Three types of Integration

1) Estimation
   RAM/Trapezoid
2) Geometry
3) Antiderivatives

2nd: Methods for Antiderivative

1) Straight forward
2) Substitution
3) By parts (after AP test)

3rd: When using the Antiderivative Method

Ask your ....

a) Can I do it straight forward meth

b) if not, simplify then see if you can do it straight forward

c) if not,
Do I have a function and that's fnct. derivative?

1) $\int \tan x \sec^2 x dx$

$u = \tan x$

$(dx)\frac{du}{dx} = \sec^2 x (dx)$

$du = \sec^2 x\, dx$

$\int u\, du$

$= \frac{u^2}{2} + C$

$= \frac{(\tan x)^2}{2} + C$

2) $\int x^4 (7 + x^5)^3 dx$

$u = 7 + x^5$

$du = 5x^4 dx$

$\frac{1}{5} du = x^4 dx$

$\int u^3 \cdot \frac{1}{5} du$

$\frac{1}{5} \int u^3 du$

$= \frac{1}{5} \cdot \frac{u^4}{4} + C$

$= \frac{1}{20} u^4 + C$

$= \frac{1}{20} (7 + x^5)^4 + C$

3) $\int r(1-r^2)^{1/2}\,dr$

$$\left(\begin{aligned} u &= 1-r^2 \\ du &= -2r\,dr \\ -\tfrac{1}{2}du &= r\,dr \end{aligned}\right)$$

$$\int u^{1/2}\cdot\left(-\tfrac{1}{2}\right)du$$

$$-\tfrac{1}{2}\int u^{1/2}\,du$$

$$= -\frac{1}{2}\cdot\frac{2}{3}u^{3/2}+C$$

$$= -\frac{1}{3}(1-r^2)^{3/2}+C$$

4) $\int \frac{9r^2}{\sqrt{1-r^3}}\,dr$

$$u = 1-r^3$$

$$(-3)\,du = -3r^2\,dr\cdot(-3)$$

$$-3\,du = 9r^2\,dr$$

$$\int \frac{-3}{\sqrt{u}}\,du$$

$$= -3\int u^{-1/2}\,du$$

$$= -3\cdot\frac{2}{1}u^{1/2}+C$$

$$= -6(1-r^3)^{1/2}+C$$

5) $\int \frac{\sin(2t+1)}{\cos^2(2t+1)}\,dt$

$$\left(\begin{aligned} u &= \cos(2t+1) \\ du &= -2\sin(2t+1)\,dt \\ -\tfrac{1}{2}du &= \sin(2t+1)\,dt \end{aligned}\right)$$

$$= \int \frac{1}{u^2}\cdot -\frac{1}{2}\,du$$

$$= -\frac{1}{2}\int \frac{1}{u^2}\,du$$

$$= -\frac{1}{2}\int u^{-2}\,du$$

$$= -\frac{1}{2}\,\frac{u^{-1}}{-1}+C$$

$$= \frac{1}{2}(\cos(2t+1))^{-1}+C$$

$$= \frac{\sec(2t+1)}{2}+C$$

6) $\int 8(y^4+4y^2+1)^2(y^3+2y)\,dy$

$$\left(\begin{aligned} u &= y^4+4y^2+1 \\ du &= (4y^3+8y)\,dy \\ du &= 4(y^3+2y)\,dy \\ 8\cdot\tfrac{1}{4}\,du &= 8(y^3+2y)\,dy \end{aligned}\right)$$

$$2\,du = 8(y^3+2y)\,dy$$

$$= \int u^2\cdot 2\,du$$

$$= 2\int u^2\,du$$

$$= 2\cdot\frac{u^3}{3}+C$$

$$= \frac{2}{3}(y^4+4y^2+1)^3+C$$

Definite Integrals

7) $\int_{2\pi}^{3\pi} 3\cos^2 x \sin x dx$

$3\int_{2\pi}^{3\pi} \cos^2 x \sin x dx$

$u = \cos x$
$du = -\sin x dx$
$-du = \sin x dx$

for new limits
$u(2\pi) = \cos(2\pi) = 1$
$u(3\pi) = \cos(3\pi) = -1$

$-3\int_{1}^{-1} u^2 du$

$= \cancel{-3} \cdot \frac{u^3}{\cancel{3}} \Big|_{1}^{-1}$

$= u^3 \Big|_{-1}^{1}$

$= (1)^3 - (-1)^3$

$= 2$

8) $\int_0^1 \sqrt{5x+4}dx$

new limits
$5(0)+4 = 4$
$5(1)+4 = 9$

$u = 5x+4$
$du = 5dx$
$\frac{1}{5}du = dx$

$\frac{1}{5}\int_4^9 u^{1/2} \cdot du$

$= \frac{1}{5} \cdot \frac{2}{3} u^{3/2} \Big|_4^9$

$= \frac{2}{15}(9)^{3/2} - \frac{2}{15}(4)^{3/2}$

$= \frac{2}{15}(3^2)^{\frac{3}{2}} - \frac{2}{15}(2^2)^{\frac{3}{2}}$

$= \frac{2}{15} \cdot 3^3 - \frac{2}{15} \cdot 2^3$

$= \frac{38}{15}$

Solve

9) $\frac{dy}{dx} = -2(y-3)$

$\frac{(\cancel{dx})}{y-3} \frac{dy}{\cancel{dx}} = -2\cancel{(y-3)} \frac{(dx)}{\cancel{y-3}}$

$\int \frac{1}{y-3} dy = -2\int dx$

$\ln|y-3| = -2x + C$

$e^{-2x+C} = y-3$

$e^{-2x+C} + 3 = y$

$y = e^{-2x+C} + 3$

$y = e^{-2x} \cdot e^{C} + 3$

$y = Ce^{-2x} + 3$

$x^m \cdot x^n = x^{m+n}$