Important Things to Know
Chapter 7 Test

Integration
Straight forward
Substitution
Rewrite

Growth & Decay
Write differential equations
Solve differential equations
Answer questions about k, t, etc.
Newton's Formula ← memorize

Slope Fields
Matching
Graph
Solve a differential equation with an initial condition

Differential Equations

1) The relative growth rate of the population of Lynton is 0.047 and its current population is P(0) = 83,400.

a) Write a *differential equation* for the population.

$$\frac{dP}{dt} = 0.047P$$

b) Find a formula for the population P in terms of t.

$$\frac{dP}{dt} = 0.047P$$

$$\int \frac{1}{P}\,dP = \int 0.047\,dt$$

$$\ln|P| = 0.047t + C$$

$$e^{0.047t+C} = P$$

$$P = Ce^{0.047t}$$

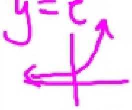
$$P(0) = 83{,}400$$

$$83{,}400 = Ce^{0.047(0)}$$

$$83{,}400 = C$$

$$\boxed{P(t) = 83{,}400e^{0.047t}}$$

$y = e^x$



2) Suppose the population of an ant colony increases at a rate proportional to the amount present initially.

a) Write a *differential equation* for the situation.

$$\frac{dP}{dt} = kP$$

b) If there are 2000 ants present initially and the population quadruples in 5 days, separate the differential equation in part (a) and write an equation for the population of the ant colony at any time *t*.

$$\int \frac{dP}{P} = \int k\,dt$$

$$\ln|P| = kt + c$$

$$e^{kt+c} = P$$

$$P = Ce^{kt}$$

$$P(0) = 2000$$

$$2000 = Ce^{k(0)}$$

$$C = 2000$$

$$P(t) = 2000e^{kt}$$

Find k

$$8000 = 2000e^{5k}$$

$$4 = e^{5k}$$

$$\ln 4 = \ln e^{5k}$$

$$\ln 4 = 5k$$

$$k = \frac{\ln 4}{5}$$

$$P(t) = 2000e^{\frac{\ln 4}{5}t}$$

3) If the half-life of a radioactive substance is 8 days, how much of a 10 gram sample will be left after 6 days?

$$5 = 10e^{k(8)}$$

$$\frac{1}{2} = e^{8k}$$

$$\ln\left(\frac{1}{2}\right) = \ln\left(e^{8k}\right)$$

$$\ln\left(\frac{1}{2}\right) = 8k$$

$$\frac{\ln\left(\frac{1}{2}\right)}{8} = k$$

$$P(t) = 10e^{\frac{\ln\left(\frac{1}{2}\right)}{8}t}$$

$$P(6) = 10e^{\frac{\ln\left(\frac{1}{2}\right)}{8}(6)}$$

$$P(6) \approx 5.946 \text{ grams}$$

4) Let P(t) represent the number of wolves in a population at time t years, when $t \geq 0$. The population P(t) is increasing at a rate directly proportional to 800 – P(t), where the constant of proportionality is k.

a) If P(0) = 500, find **P(t)** in terms of t and k.

$$\frac{dP}{dt} = k(800 - P)$$

$$\int \frac{1}{800-P}\,dP = k\int dt$$

$$-\ln|800-P| = kt + C$$

$$\ln|800-P| = -kt + C$$

$$e^{-kt+C} = 800 - P$$

$$P = 800 - Ce^{-kt}$$

$$P(0) = 500$$

$$500 = 800 - Ce^{-k(0)}$$

$$500 = 800 - C$$

$$C = 300$$

$$P(t) = 800 - 300e^{-kt}$$

b) If P(2) = 700, find k.

$$700 = 800 - 300e^{-k(2)}$$

$$-100 = -300e^{-2k}$$

$$\frac{1}{3} = e^{-2k}$$

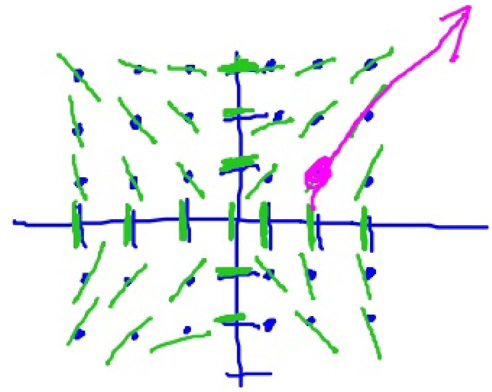
$$\ln\left(\frac{1}{3}\right) = -2k$$

$$\boxed{k = \frac{\ln\left(\frac{1}{3}\right)}{-2}}$$

16) For $\frac{dy}{dx} = \frac{x}{y}$

a) Draw the slope field for the given graph

b) Sketch the curve if y(2) = 1



c) Solve the differential equation

$$\frac{dy}{dx} = \frac{x}{y}$$

$$\int y\,dy = \int x\,dx$$

$$\frac{y^2}{2} = \frac{x^2}{2} + C$$

$$\sqrt{y^2} = \sqrt{x^2 + C}$$

$$y = \pm\sqrt{x^2 + C}$$

$y(2) = 1$

$(2,1)$

$$(1)^2 = \left(\sqrt{(2)^2 + C}\right)^2$$

$$1 = 4 + C$$

$$C = -3$$

$$y = \sqrt{x^2 - 3}$$

** Must know all information from Slope Fields Worksheets

5) Suppose the population of Boomtown increases at a rate proportional to the amount present initially.

a) Write a *differential equation* for the situation.

b) If there are 800 people present initially and the population triples in 20 hrs., *separate* the differential equation in part (a) and write and equation for the population of Boomtown at any time t.

c) What will the population be in 3 days?

d) When will the population reach 8000?

6) Suppose the rabbit population of Tulare County grows exponentially. If the number of rabbits was estimated at 30,000 in 1975 and at 45,000 in 1978, approximate the rabbit population in 1981.
(round your answer to the nearest rabbit)