Section 8.1: Integral as Net Change

1. An ant moves along the x-axis with velocity $v(t) = t^2 - 6t + 8$ [0, 4] $v(t)$ is measured in ft/sec.

a) Determine when the ant is moving right, left, and stopped

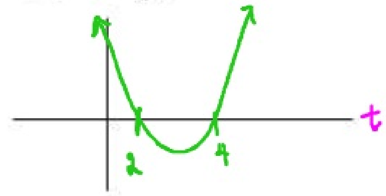
$\underline{v(t) = 0}$

$t^2 - 6t + 8 = 0$

$(t-2)(t-4) = 0$

$t = 2, 4$

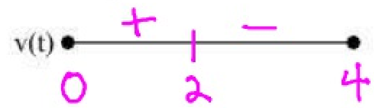
Graph of $v(t)$



right: $[0, 2)$ b/c $v > 0$

left: $(2, 4)$ b/c $v < 0$

stopped: $t = 2, 4$ seconds b/c $v = 0$



b) Displacement of the ant over the first 3 seconds

note

$$\int_0^3 v(t)\,dt$$

$$S(t)\Big|_0^3$$

$$\int_0^3 (t^2 - 6t + 8)\,dt$$

$$\frac{t^3}{3} - \frac{6t^2}{2} + 8t \Big|_0^3$$

$$\left[\frac{(3)^3}{3} - \frac{6(3)^2}{2} + 8(3)\right] - \left[0 - 0 + 0\right]$$

$$= 6 \text{ ft.}$$

c) The position of the ant is given by the function $s(t)$.

At $t = 0$ the ant's initial position is -5 *(Note: $s(0) = -5$)*

Find the position of the ant when t = 1 second

$s(1) = S(0) + \int_0^1 v(t)\,dt$

$= -5 + \int_0^1 (t^2 - 6t + 8)\,dt$

$= -5 + 5.\overline{3}$

$S(1) = .\overline{3}$ ft.

Find the position of the ant when t = 3 seconds

$s(3) = S(0) + \int_0^3 v(t)\,dt$

$= -5 + 6$

$= 1$ ft.

Find the position of the ant when t = 2 seconds

$s(2) = S(0) + \int_0^2 v(t)\,dt$

$= -5 + 6.\overline{6}$

$S(2) = 1.\overline{6}$ ft.

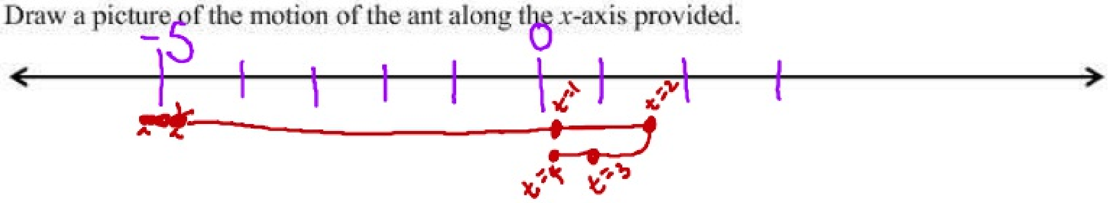
Find the position of the ant when t = 4 seconds

$s(4) = S(0) + \int_0^4 v(t)\,dt$

$= -5 + 5.333$

$S(4) = 0.\overline{3}$

Draw a picture of the motion of the ant along the x-axis provided.



d) Determine the total distance the ant traveled over first 4 seconds

analytically *(without calculator)*

$$= \int_0^2 v(t)\,dt - \int_2^4 v(t)\,dt$$

$$= \int_0^2 (t^2-6t+8)\,dt + \int_4^2 (t^2-6t+8)\,dt$$

$$= \left(\frac{t^3}{3}-3t^2+8t\right)\Big|_0^2 + \left(\frac{t^3}{3}-3t^2+8t\right)\Big|_4^2$$

$$= 8\text{ ft.}$$

with calculator

$$= \int_0^4 |v(t)|\,dt$$

$$= 8\text{ ft.}$$

******note on your assignment – you must do the antiderivatives by hand on problems 1, 3, 5, 7.***

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2. A helicopter moves along the y-axis with velocity $v(t) = 6\sin(3t)$ $\left[0, \frac{\pi}{2}\right]$ $v(t)$ is measured in ft/sec.

a) Determine when the helicopter is moving up, down, and stopped

$$v(t) = 0$$

$$6\sin(3t) = 0$$

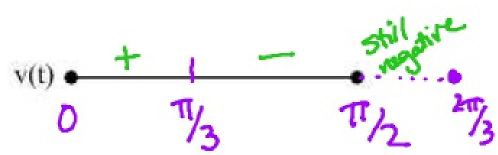
$$\sin(3t) = 0$$

$$3t = x$$

$$\sin(x) = 0$$

$$x = 0, \pi, 2\pi \qquad 3t = 0, \pi, 2\pi$$

$$t = 0, \tfrac{\pi}{3}, \tfrac{2\pi}{3}$$



up: $[0, \pi/3)$ b/c $v > 0$
down: $(\pi/3, \pi/2]$ b/c $v < 0$
stopped: $t = \pi/3$ seconds b/c $v = 0$

b) Displacement of the helicopter over the first $\frac{\pi}{2}$ seconds

$$\int_0^{\pi/2} 6\sin(3t)\,dt$$

$$= -2\cos(3t)\Big|_0^{\pi/2}$$

$$= \left[-2\cos\left(3\cdot\frac{\pi}{2}\right)\right] - \left[-2\cos(3\cdot 0)\right]$$

$$= -2\cdot 0 + 2\cdot 1$$

$$= 2\text{ ft.}$$

$s' = v$
$s'' = v' = a$

c) The position the helicopter at any given time is $s(t)$.
At $t = 0$ the helicopter's initial position is 4 *(Note: $s(0) = 4$)*

Find the *position* of the helic. when $t = \frac{\pi}{3}$ secs.

$s(\frac{\pi}{3}) = S(0) + \int_0^{\pi/3} 6\sin(3t)\,dt$

$= 4 + 4$

$S(\frac{\pi}{3}) = 8$ ft.

Find the *position* of the helic. when $t = \frac{\pi}{2}$ secs.

$s(\frac{\pi}{2}) = S(0) + \int_0^{\pi/2} 6\sin(3t)\,dt$

$= 4 + 2$

$S(\frac{\pi}{2}) = 6$ ft.

d) Determine the total distance the helicopter traveled over first $\frac{\pi}{2}$ seconds

non-calc

$\int_0^{\pi/3} 6\sin(3t)\,dt - \int_{\pi/3}^{\pi/2} 6\sin(3t)\,dt$

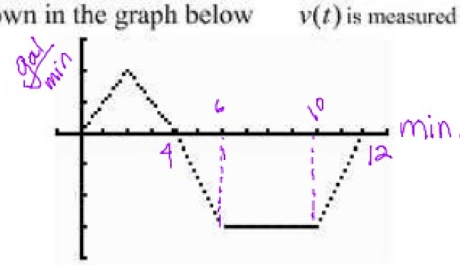
calc.

$\int_0^{\pi/2} |v(t)|\,dt$

≈ 6.000001837

≈ 6.000 ft.

3. A tank is filled and drained of chocolate milk at a rate determined by the velocity function $v(t)$ as shown in the graph below $v(t)$ is measured in gal/min.



a) Determine when the tank is filling up, draining, or stopped.

filling up: $(0, 4)$ b/c $v > 0$
draining: $(4, 12)$ b/c $v < 0$
stopped: $t = 0, 4, 12$ min. b/c $V = 0$

b) Determine the change in the amount of chocolate milk in the tank over the first 4 mins.

$\int_0^4 v(t)\,dt$

$= \frac{1}{2}(4)(2)$

$= 4$ gallons

Determine the change in the amount of chocolate milk in the tank over the first 6 mins.

$\int_0^6 v(t)\,dt$

$= \frac{1}{2}(4)(2) - \frac{1}{2}(2)(3)$

$= 4 - 3$

$= 1$ gallon

c) If the tank starts out with 20 gallons of chocolate milk at t = 0 mins. The function to represent the amount of chocolate milk in the tank at any given time is G(t) ∴ **G(0) =** 20 **gallons.**

Determine the amount of chocolate milk in the tank at t = 4, 6 and 12.

$G(4) = G(0) + \int_0^4 v(t)\,dt$

$= 20 + 4$

$G(4) = 24$ gallons

$G(6) = G(0) + \int_0^6 v(t)\,dt$

$= 20 + 1$

$G(6) = 21$ gallons

$G(12) = G(0) + \int_0^{12} v(t)\,dt$

$= 20 - 14$

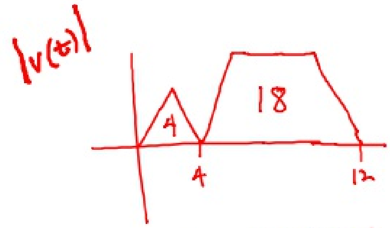
$G(12) = 6$ gallons

d) Determine the total amount of chocolate milk flowing in and out of the tank over the first 12 mins.

$\int_0^4 v(t)\,dt - \int_4^{12} v(t)\,dt$

$= 4 - (-18)$

$= 22$ gallons



$\frac{1}{2}(4)(2)$ $= 4$

$\frac{1}{2}(3)(8+4)$ $= 18$

$\int_0^{12} |v(t)|\,dt$

$= 4 + 18$

$= 22$ gallons