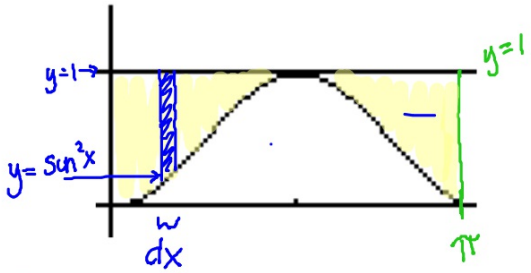
Area Between Curves
Section 8.2

Find the area trapped between the curves

1) $y_2 = \sin^2 x \qquad [0,\pi]$

$y_1 = 1$



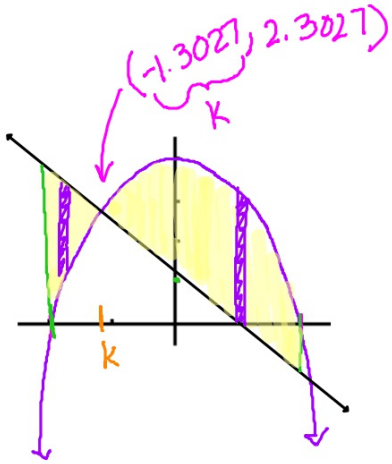
$$\text{Area} = \int_0^{\pi} (\text{top curve} - \text{bottom curve})\,dx$$

$$\text{Area} = \int_0^{\pi} (1 - \sin^2 x)\,dx$$

$$\text{Area} \approx 1.5708$$

2) $y_1 = 4 - x^2 \qquad [-2,2]$

$y_2 = 1 - x$



$$\text{Area} = \int_{-2}^{k} \Big(\underbrace{(1-x)}_{\text{top}} - \underbrace{(4-x^2)}_{\text{bottom}} \Big)\, dx + \int_{k}^{2} \Big(\underbrace{(4-x^2)}_{\text{top}} - \underbrace{(1-x)}_{\text{bottom}} \Big)\, dx$$

$$\text{Area} = \int_{-2}^{k} (y_2 - y_1)\, dx + \int_{k}^{2} (y_1 - y_2)\, dx$$

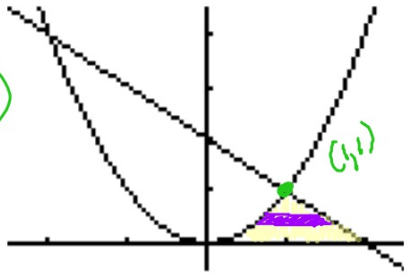
$\text{Area} \approx 8.645$

3) $y = x^2 \rightarrow x = \pm\sqrt{y}$

$x = \sqrt{y}$ (First quad only needed)

$x + y = 2$ $\quad x = 2 - y$

$x - axis$ $\quad y = 0$



pt. of intersection

$(\sqrt{y})^2 = (2-y)^2$

$y = (2-y)^2$

$y = 4 - 4y + y^2$

$0 = y^2 - 5y + 4$

$(y-4)(y-1) = 0$

$y = 4, 1$

$$\text{Area} = \int [\text{big right} - \text{small left}]\, dy$$

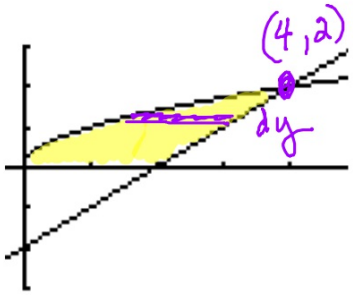
$$\text{Area} = \int_0^1 [(2-y) - (\sqrt{y})]\, dy$$

$$\text{Area} \approx 0.8333$$

4) $y = x - 2$ $x = y + 2$

$y = \sqrt{x}$ $x = y^2$

$x - axis$



pt of int.

$y + 2 = y^2$

$0 = y^2 - y - 2$

$0 = (y-2)(y+1)$

$y = 2, -1$

$\text{Area} = \int_0^2 [(y+2) - (y^2)]\,dy$

$\text{Area} \approx 3.333$