Important things to know for the 1st Quarter Final Exam

Domain of any function

Limits

- At a point
  - o Algebraically
    - Plug in
    - Factor, cancel, plug-in
  - o Graphically (from tables or graphs)
- L Hopital's Rule $\lim_{x\to 5} \frac{x^2-2x-15}{x-5} = \lim_{x\to 5} \frac{2x-2}{1} = 2(5)-2 = \boxed{8}$
- $\lim_{x\to 0} \frac{\sin x}{x} = 1$
- Limits involving Infinity $\lim_{x\to\infty} f(x) = a$ and/or $\lim_{x\to -\infty} f(x) = b$, then the H.A. $y = a$ and/or b
  - Vertical Asymptotes
  - End Behavior Asymptotes (Horizontal)

    <u>3 Cases</u>

    Case 1: Highest Exponent in the numerator

    Case 2: Highest Exponent the same in the numerator & denominator

    Case 3: Highest Exponent in the denominator
- Comparing relative magnitudes of a function and their rates of change (x^2 vs. e^x)

  $\lim_{x\to\infty} (e^x+2x)$ vs. $\lim_{x\to -\infty} (e^x+2x)$

Continuity

- Definition
  - o $\lim_{x\to a} f(x)$ exists
  - o $f(a)$ exists
  - o $\lim_{x\to a} f(x) = f(a)$
- Types of discontinuity Hole, jumps, asymptotes
- How to make a piecewise function continuous

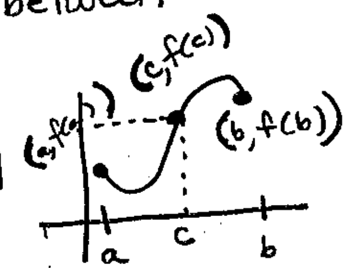
  set the pieces equal, then plug-in the x-Value

  ex. $f(x) = \begin{cases} 3x+2 & x<1 \\ ax & x \geq 1 \end{cases}$

IVT – Intermediate Value Theorem:

A function $y=f(x)$ that is continuous on a closed interval $[a,b]$ takes on every value between $f(a)$ and $f(b)$

<u>In other words</u>, If y_0 is between $f(a)$ and $f(b)$ Then $y_0 = f(c)$ for some c in $[a,b]$



Differentiability

- Types of non-differentiable points
  - cusp / corner (if you zoom-in on the point of concern the graph should look like a line (local linearity))
  - jump
  - ~~infinite~~ vertical tangent
- Check if a function is / is not differentiable at a point

  1st: Is the function ~~differentiable~~ continuous at the point?

  2nd: Is the $\lim_{x \to c^-} f'(x) = \lim_{x \to c^+} f'(x)$ Are the slopes the same on the left and right side of the point?

Derivatives

- Derivative presented graphically, numerically, and analytically

  next quarter

Rates of Change = Slope = Derivative

- average rate of change

  $$\frac{f(b)-f(a)}{b-a}$$

- instantaneous rate of change $\lim_{h \to 0} \frac{f(x+h)-f(x)}{(x+h)-(x)} = f'(x)$

- recognizing the definition of a derivative *(difference quotient)*

  ex. $\lim_{h \to 0} \frac{[(2+h)^2+3(2+h)]-[(2)^2+3(2)]}{(2+h)-2} = \frac{d}{dx}\left(x^2+3x\right)(2)$

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- Understand the relationship between continuity and differentiability

  For a function to be differentiable at ~~any~~ a pt. the function must be continuous at that point

Equations

- lines $y - y_1 = m(x - x_1)$

  $y = mx + b$

Space →
- tangent / normal

Derivatives: 1st, 2nd, etc.

- power rule $\frac{d}{dx}(x^n) = n x^{n-1}$
- product rule $\frac{d}{dx}(f \cdot g) = fg' + gf'$
- quotient rule $\frac{d}{dx}\left(\frac{f}{g}\right) = \frac{g \cdot f' - f \cdot g'}{g^2}$
- trig.

$f(x)$	$f'(x)$
$\sin x$	$\cos x$
$\cos x$	$-\sin x$
$\tan x$	$\sec^2 x$
$\cot x$	$-\csc^2 x$
$\csc x$	$-\csc x \cot x$
$\sec x$	$\sec x \tan x$

- chain rule
  $$\frac{d}{dx}[f(g(x))] = f'(g(x)) \cdot g'(x) = \frac{du}{dx}$$
- exponential
  $$\frac{d}{dx}[a^u] = a^u \cdot \ln a \cdot \frac{du}{dx}$$
- logarithms
  $$\frac{d}{dx}[\log_a u] = \frac{1}{u} \cdot \frac{1}{\ln a} \cdot \frac{du}{dx}$$
- ~~implic~~it differentiation
  logarithmic $y = x^x$
- implicit differentiation $x + \sin x \cdot y - y^3 = 4$
- Derivative of a function's inverse
  if g and f are inverses of each other then $g(f(x)) = x$ etc.

Derivative as a function

- Corresponding characteristics of f and f'

  Given a picture
  - of f, draw f'
  - of f', draw f