Sections 6.3 and 6.4: *(part 2)*

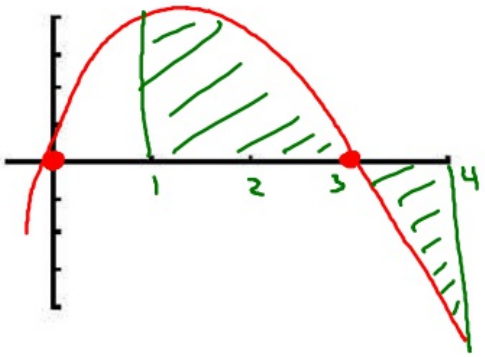
Integration vs. *Total* Area

Net Area Gross Area

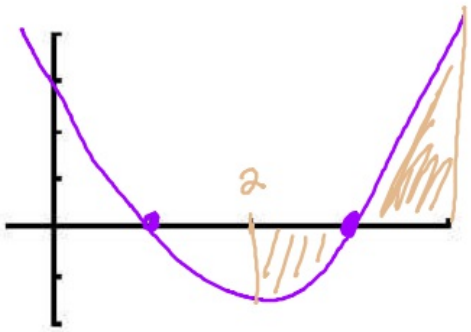
Integral = Area

Graph "y" and shade the region that will be integrated

1) $y = -x(x-3)$ on $[1, 4]$



2) $y = (x-1)(x-3)$ on $[2, 4]$



Find:

a) The ***value*** of the integral — Net Area

b) ***Total area*** of the region — Gross Area

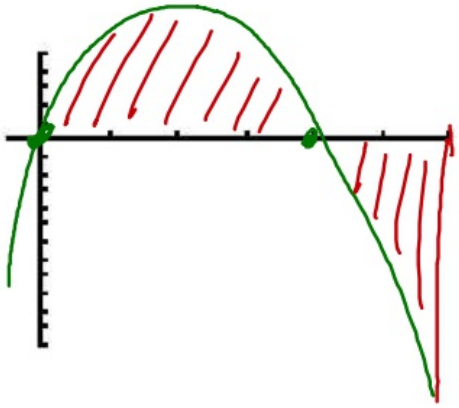
Note: students may use a calculator to evaluate

3) $y = 4x - x^2$ on $[0, 6]$

Evaluate the *Integral*

$$\int_0^6 (4x - x^2)\,dx$$

$= 0$ (using the calculator)



Evaluate the *Total Area*

$$\text{Total Area} = \int_0^4 [\underbrace{(4x-x^2)}_{\text{top}} - \underbrace{(0)}_{\text{bottom}}]\,dx + \int_4^6 [\underbrace{(0)}_{\text{top}} - \underbrace{(4x-x^2)}_{\text{bottom}}]\,dx$$

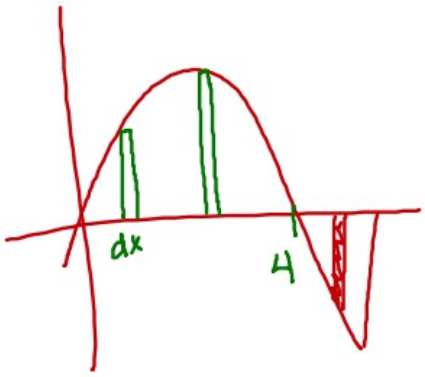
$$\text{Total Area} = \int_0^4 (4x-x^2)\,dx - \int_4^6 (4x-x^2)\,dx$$

$= 10.\overline{6} - (-10.\overline{6})$

$\text{Total Area} = 21.3$

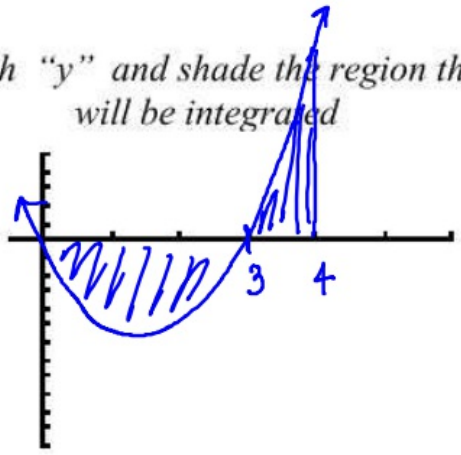
After evaluating *the integral . . .*

- *Draw the representative rectangles*
- *Factor out the negative sign*
- *Talk about why the second integral has a negative in front*



4) $y = x^2 - 3x$ on $[0, 4]$

Graph "y" and shade the region that will be integrated



Evaluate the *Integral*

$$\int_0^4 (x^2 - 3x)\,dx$$

$$= -2.\overline{6}$$

Evaluate the *Total Area*

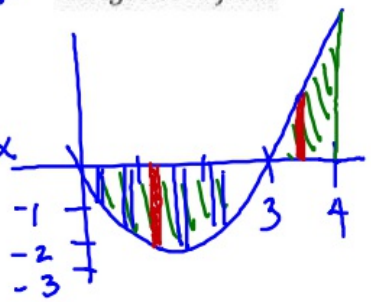
After evaluating *the integral . . .*

- *Draw the representative rectangles*
- *Factor out the negative sign*
- *Talk about why the first integral has a negative in front*

$$\text{Total Area} = \int_0^3 \left[(0) - (x^2 - 3x)\right] dx + \int_3^4 \left[(x^2 - 3x) - (0)\right] dx$$

$$= -\int_0^3 (x^2 - 3x)\,dx + \int_3^4 (x^2 - 3x)\,dx$$

$$= -(-4.5) + 1.8\overline{3}$$



$$\text{Total Area} = 6.\overline{3}$$

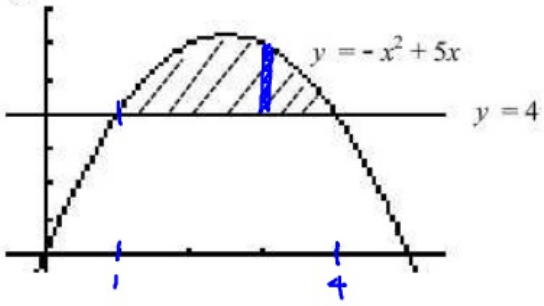
$$0 - (x^2 - 3x)$$

$$-x^2 + 3x$$

$f(x)\,h$

Find the *Total Area* of the shaded region

5)



To find their intersections

$y_1 = y_2$

$-x^2 + 5x = 4$

$0 = x^2 - 5x + 4$

$0 = (x-1)(x-4)$

$x = 1, 4$

$$\text{Total Area} = \int_1^4 \left[\overbrace{(-x^2+5x)}^{\text{top}} - \overbrace{(4)}^{\text{bottom}} \right] dx$$

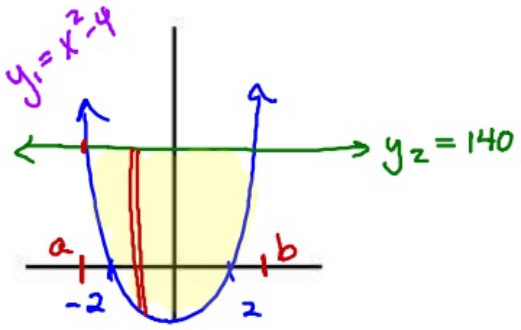
$$\text{Total Area} = 4.5$$

Find the *Area* trapped between the curves

6) $y_1 = x^2 - 4$ and $y_2 = 140$

$a =$

$b =$

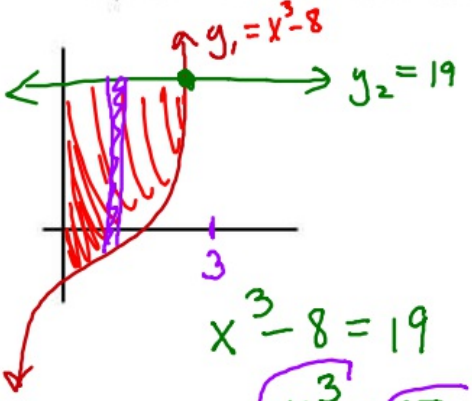


$$\text{Total Area} = \int_a^b [y_2 - y_1]\, dx$$

$$= 2304$$

Set-up only

7) $y = x^3 - 8$ and $y = 19$ *(right side of y-axis only)*



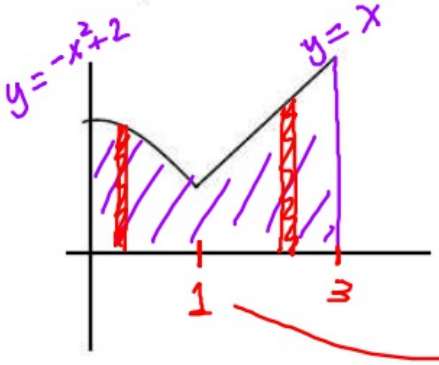
$$\text{Total Area} = \int_0^3 \left[\underbrace{(19)}_{\text{top}} - \underbrace{(x^3 - 8)}_{\text{bottom}} \right] dx$$

$x^3 - 8 = 19$

$\sqrt[3]{x^3} = \sqrt[3]{27}$

$x = 3$

8) $y = -x^2 + 2$ and $y = x$ on $[0, 3]$ *(1^{st} Quadrant only)*



pt of int

$-x^2 + 2 = x$

$0 = x^2 + x - 2$

$0 = (x - 1)(x + 2)$

$x = 1, -2$

$x = 1$ only needed

$$\text{Total Area} = \int_0^1 \left[\underbrace{(-x^2+2)}_{\text{top}} - \underbrace{(0)}_{\text{bottom}} \right] dx + \int_1^3 \left[\underbrace{(x)}_{\text{top}} - \underbrace{(0)}_{\text{bottom}} \right] dx$$

$= 5.\overline{6}$