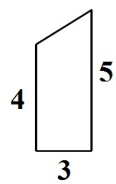
Sections 5.5: Trapezoid Rule
and
Fundamental Theorem of Calculus (part I)

Write the formula to evaluate the **Area of a Trapezoid**

$$A = \frac{1}{2}h(b_1 + b_2)$$

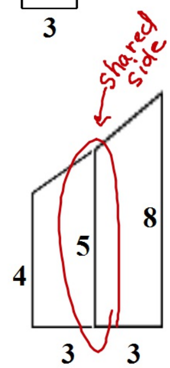
Determine the area of the trapezoid



$$A = \frac{1}{2}(3)(4+5)$$

$$A = \frac{27}{2}$$

Determine the area of the trapezoids



$$A = \frac{1}{2}(3)(4+5) + \frac{1}{2}(3)(5+8)$$

shared side

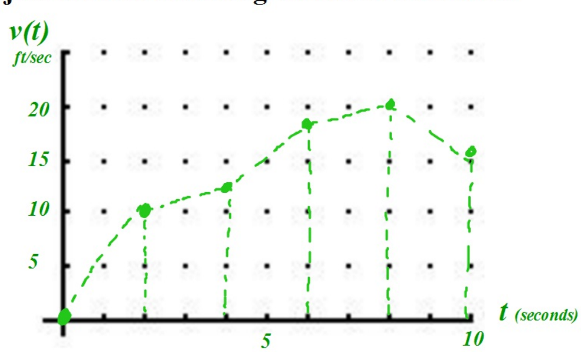
$$A = \frac{1}{2}(3)\left[4 + (2)(5) + 8\right]$$

$$A = \frac{3}{2}\left[22\right]$$

$$A = 33$$

1) A moving object had the following velocities recorded.

t	$v(t)$
0	0
2	10
4	12
6	18
8	20
10	16



Estimate the distance traveled over the first 10 seconds using a Trapezoidal approximation.
(using 5 intervals)

Total distance traveled $= \int_0^{10} v(t)\, dt$

shared side

$$\approx \frac{1}{2}(2)\left[v(0) + 2\cdot v(2) + 2\cdot v(4) + 2\cdot v(6) + 2\cdot v(8) + v(10)\right]$$

$$\approx 1\left[0 + 2(10) + 2(12) + 2\cdot(18) + 2(20) + 16\right]$$

$\approx$ 136 ft. traved over the first 10 seconds

2) Water filling a tank had the following velocities recorded.

t	$v(t)$ gal/min
0	0
4	10
8	15
12	13
16	5
20	25
24	30

a) Estimate the gallons of water that flowed into the tank over the first 24 minutes using a Trapezoid approximation. (using 6 intervals)

$$\text{total gallons} = \int_0^{24} v(t)\,dt$$

$$\approx \frac{1}{2}(4)\left[0 + 2v(4) + 2v(8) + 2v(12) + \cdots + 2v(20) + v(24)\right]$$

$$\approx 2\left[0 + 2\cdot 10 + 2\cdot 15 + 2\cdot 13 + 2\cdot 5 + 2\cdot 25 + 30\right]$$

≈ 332 gallons over the first 24 mins.

b) Estimate the average acceleration over the *first* 12 minutes

$$\frac{v(12)-v(0)}{12-0} = \frac{13-0}{12} = \frac{13\ \text{gal/min}}{12\ \text{min}} = \frac{13}{12}\ \text{gal/min}^2$$

c) Estimate the acceleration at 10 minutes

$$\frac{v(12)-v(8)}{12-8} = \frac{13-15}{4} = \frac{-2}{4} = -\frac{1}{2}\ \text{gal/min}^2$$

d) When is the acceleration > 0 ? $(0,8)\cup(16,24)$ b/c $a = v' > 0$

v is increasing

continued . . . **2) Water filling a tank**

e) Determine the average velocity for the first 24 minutes using the trapezoidal approximation found in part "a".

$$\frac{1}{b-a}\int_a^b f(x)\,dx$$

Ave. Value of a function

$$\text{Ave. value} = \frac{1}{24-0}\int_0^{24} v(t)\,dt$$

$$\approx \frac{1}{24}(332)\ \frac{\text{gallons}}{\text{mins}}$$

$$\approx 13.833\ \frac{\text{gallons}}{\text{min}}$$

Explain the meaning of your answer in part "e".

The average rate that the water is flowing into the tank for the first 24 minutes is about 13.833 gal/min.

*Typically approximation methods, such as RAM and Trapezoid Method are used with **tables** of values, but not exclusively.*

3) #2 on page 316 $\displaystyle\int_0^2 x^2\,dx$

a) Use the Trapezoid Rule with $n = 4$ to approx. the value of the integral.

$$\text{Area} = \int_0^2 x^2\,dx$$

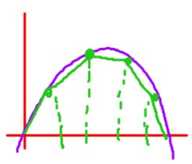
$$\approx \frac{1}{2}\left(\frac{1}{2}\right)\left[f(0) + 2\cdot f\left(\tfrac{1}{2}\right) + 2\cdot f(1) + 2\cdot f(1.5) + f(2)\right]$$

$$\approx \frac{1}{4}\left[0 + 2\left(\tfrac{1}{4}\right) + 2(1) + 2\cdot\left(\tfrac{9}{4}\right) + 4\right]$$

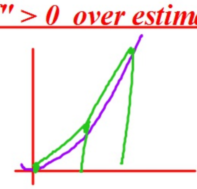
$$\approx \frac{11}{4} \text{ or } 2.75$$

***NOTE:** It is possible to determine if an answer is an overestimation, underestimation or exact by determining the concavity of the function.*

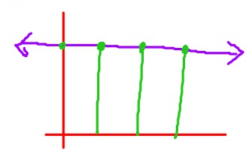
$f'' < 0$ under estimation



$f'' > 0$ over estimation



$f'' = 0$ exact estimate



continued

3) #2 on page 316 $\displaystyle\int_0^2 x^2\,dx$

b) Use the concavity of the function to predict whether the approximation is an overestimate or an underestimate.

$f(x) = x^2$

$f'(x) = 2x$

$f''(x) = 2$

Since $f'' > 0$, the trapezoidal estimate of $\frac{11}{4}$ in part "a" is an over estimate.

c) Find the integral's exact value to check your answer in part "b".

$$\int_0^2 x^2\,dx$$

$$= \frac{1}{3}x^3\Big|_0^2$$

$$= \left[\frac{1}{3}(2)^3\right] - \left[\frac{1}{3}(0)^3\right]$$

$$= \frac{8}{3} - 0$$

$$= \frac{8}{3}$$

$$= 2.\overline{6}$$

$2.\overline{6} < 2.75$

We are right!

Fundamental Theorem of Calculus (Parts I and II)

Fundamental Theorem of Calculus **(Part II)**

$$\int_a^b f'(x)\,dx \qquad \int_a^b f(x)\,dx \qquad \int_a^b v(t)\,dt \qquad \int_a^b a(t)\,dt$$

$= f(x)\Big|_a^b$ $\qquad = F(b)-F(a)$ $\qquad = S(b)-S(a)$ $\qquad = V(b)-V(a)$

$= f(b)-f(a)$

1) **Simplify** (using antiderivatives)

$$\frac{d}{dx}\left[\int_5^x (2t-1)\,dt\right]$$

$= \frac{d}{dx}\left[t^2-t\Big|_5^x\right]$

$= \frac{d}{dx}\left[(x^2-x)-(5^2-5)\right]$

$\frac{d}{dx}\left[x^2-x-20\right]$

$= 2x-1$

Write a rule from what you learned in the previous problem

$$\frac{d}{dx}\left[\int_A^x f(t)\,dt\right] = f(x)$$

2) **Simplify** (using your rule)

$$\frac{d}{dx}\left[\int_7^x (5t^2+4)\,dt\right] = 5x^2+4$$

3) **Simplify** (using antiderivatives)

$$\frac{d}{dx}\left[\int_5^{3x^2}(2t-1)\,dt\right] = \frac{d}{dx}\left[t^2 - t \Big|_5^{3x^2} \right]$$

$$= \frac{d}{dx}\left[\left[(3x^2)^2 - (3x^2)\right] - \left[(5)^2 - (5)\right]\right]$$

$$= \frac{d}{dx}\left[9x^4 - 3x^2 - 20\right]$$

$$= 36x^3 - 6x$$

$$= 6x \cdot (6x^2 - 1)$$

$$\frac{d}{dx}(3x^2) \cdot \left[2(3x^2) - 1\right]$$

$f(t) = 2t - 1$

$$\boxed{\begin{array}{l} f(x) = 2x - 1 \\ g(x) = 3x^2 \end{array}}$$

Rule: $g'(x) \cdot f(g(x))$

Write a rule from what you learned in the previous problem

$$\frac{d}{dx}\left[\int_A^{g(x)} f(t)\,dt\right] = g'(x) \cdot f(g(x))$$

This rule represents the

Fundamental Theorem of Calculus **(part I)**

4) **Simplify using the Fundamental Theorem of Calculus (part I)**

$$\frac{d}{dx}\left[\int_4^{\sin x}(e^t + 2)\,dt\right]$$

$$= \cos x \cdot \left(e^{\sin x} + 2\right)$$

5) **Simplify using the Fundamental Theorem of Calculus (part I)**

$$\frac{d}{dx}\left[\int_{3x^4+2}^{2}(-2t-3)dt\right]$$

$$= -\frac{d}{dx}\left[\int_{2}^{3x^4+2}(2t+3)\,dt\right]$$

$$= 12x^3 \cdot \left[2(3x^4+2)+3\right]$$

$$= 12x^3\left[6x^4+\overbrace{4+3}^{7}\right]$$

$$= 72x^7 + 84x^3$$

6) **page 308 #58**

$$s = \int_0^t f(x)\,dx$$

First step ! Write down the relationships for the problem

a) What is the particle's velocity at time $t = 5$?

b) Is the acceleration of the particle at time $t = 5$ positive or negative?

c) What is particle's position at time $t = 3$?

d) At what time during the first 9 seconds does "s" have its largest value?

e) Approximately when is the acceleration zero?

f) When is the particle moving toward the orgin?

away from the origin?

g) On which side of the origin does the particle lie at time $t = 9$?