Section 6.2: Evaluating Definite Integrals using GEOMETRY

Three ways to integrate use the eraser to reveal the answers

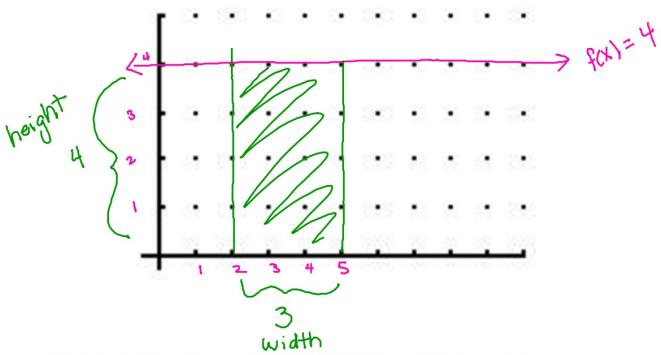
1. **Estimation - RAM (Rectangular Approximation Method)** ***or*** **Trapezoid Method**
2. **Geometry**
3. **Antiderivatives**

Integration Notation use the eraser to reveal the answers

$$\int_{a}^{b} f(x)\,dx$$

end (b) · begin (a) · height: $f(x)$ · width: dx

Graph $f(x) = 4$ **[2, 5]**



Determine the *area* created by the graph

$A = w \cdot h$
$A = (5-2)(4-0)$
$A = (3)(4)$
$A = 12$

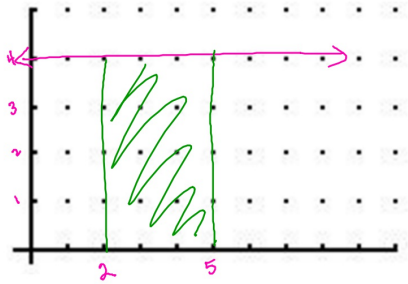
Example #1

Evaluate the integral

$$\int_{2}^{5} 4\,dx$$

height: 4 · width: dx

$= 4(3)$
$= 12$



Quick Review: **Write the formula for the *Area* of a Trapezoid**

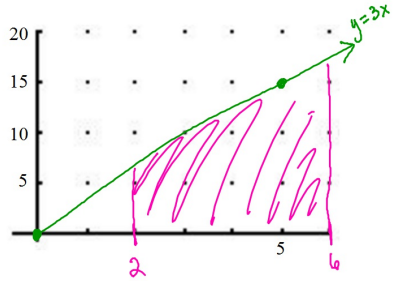
use the eraser to reveal the answer

$$A = \frac{1}{2} h\left(b_1 + b_2\right)$$

Example #2

$$\int_{2}^{6} 3x\,dx$$

$= \frac{1}{2}(\text{width})(\text{height})$
$= \frac{1}{2}(6-2)(6+18)$
$= \frac{1}{2}(4)(24)$
$= 48$

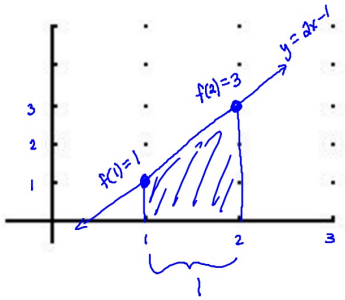


Example #3

$$\int_1^2 (2x-1)\,dx$$

$= \frac{1}{2}(1)(1+3)$

$= 2$

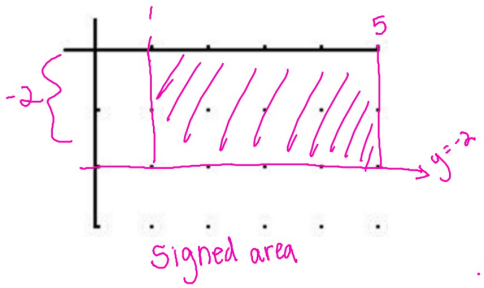


Example #4

$$\int_1^5 -2\,dx$$

$= -2(4)$

$= -8$



Quick Review: Solve for y

$x^2 + y^2 = 25$

$y^2 = 25 - x^2$

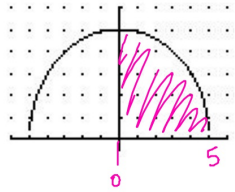
$y = \pm\sqrt{25-x^2}$

Example #5

$$\int_0^5 \sqrt{25-x^2}\,dx$$

$= \frac{1}{4}\pi(5)^2$

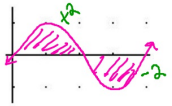
$= \frac{25}{4}\pi$



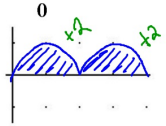
Example #6 *Memorize this for now*

$$\int_0^{\pi} \sin x\,dx = 2$$

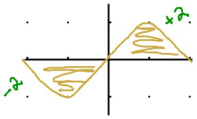
a) $\int_0^{2\pi} \sin x\,dx = 0$



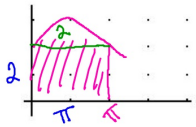
b) $\int_0^{2\pi} |\sin x|\,dx = 4$



c) $\int_{-\pi}^{\pi} \sin x\,dx = 0$



d) $\int_0^{\pi} (2+\sin x)\,dx = 2+2\pi$

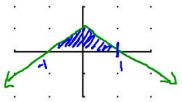


Example #7

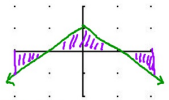
a) $\int_{-1}^{1} (1-|x|)\,dx$

$= \frac{1}{2}(2)(1)$

$= 1$



b) $\int_{-2}^{2} (1-|x|)\,dx$



$= \frac{1}{2}(2)(1) - (2)\left(\frac{1}{2}\right)(1)(1)$

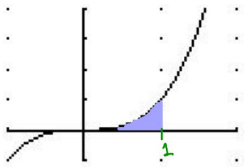
$= 1 - 1$

$= 0$

Table 6.3: Rules for Definite Integrals *Look at page 289*

Example #8

Given: $\int_0^1 x^3\,dx = \frac{1}{4}$

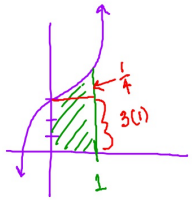


Find:

$\int_0^1 (x^3+3)\,dx$

$= \frac{1}{4} + 3$

$= 3\frac{1}{4}$



Using a graphing calculator to evaluate integrals

Math (menu)

9: fnInt(f(*x*) , *x*, begin , end)

Example #9

a) $\int_0^5 e^{-x^2}\,dx$

$fnInt(e^{-x^2}, x, 0, 5)$

note: Calculator notation is *not* acceptable for showing work on a test

$\approx$ **0.886**

b) $\int_{0.762}^{5.314} (xe^x - \sin x)\,dx$

$fnInt(y_1, x, 0.762, 5.314)$

$\approx$ **876.790**

note: you must write the 0 at the end since there are more digits after the 0.