Overview of Chapter 7

Antiderivatives

Definite

$$\int_1^2 2x\,dx$$

$$= x^2 \Big|_1^2$$

$$= (2)^2 - (1)^2$$

$$= \boxed{3}$$

Indefinite

$$\int 2x\,dx$$

$$= \boxed{x^2 + C}$$

Method for determining antiderivatives

- straight forward (may need to be simplified first)
- substitution
- by parts (after AP test)

Solving *Differential Equations*

$$\frac{dy}{dx} = \cdots$$

$$\downarrow$$

$$y = \cdots$$

Growth and Decay Problems

Exponential functions

Slope Fields

Section 7.1: Antiderivatives and Differential Equations

Review

$$\int x^n dx$$

$$= \frac{x^{n+1}}{n+1} + C$$

$$\int x^3 dx$$

$$= \frac{x^4}{4} + C$$

<u>Evaluate each indefinite integral</u>

1) $\int 2x^4 dx$

$$= 2\int x^4 dx$$

$$= 2\frac{x^5}{5} + C$$

$$= \frac{2}{5}x^5 + C$$

2) $\int \frac{1}{x} dx$

$$= \ln|x| + C$$

3) $\int \frac{2}{3x} dx$

$$= \frac{2}{3}\int \frac{1}{x} dx$$

$$= \frac{2}{3}\ln|x| + C$$

4) $\int (-x^{-3} + x - 1)dx$

$= \frac{x^{-2}}{2} + \frac{x^2}{2} - x + C$

$\boxed{\text{OR}}$ $= \frac{1}{2x^2} + \frac{x^2}{2} - x + C$

5) $\int \frac{2}{\sqrt[3]{x}}dx$

$= \int 2x^{-1/3}\,dx$

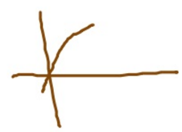
$= 2 \cdot \frac{3}{2} x^{2/3} + C$

$= 3x^{2/3} + C$

OR $= 3\sqrt[3]{x^2} + C$

Review

$\frac{d}{dx}(\sin(5x))$ = $5\cos(5x)$



6) $\int 3\cos(3x)dx$

$= \sin(3x) + c$

7) $\int \sin(5x)dx$

$= -\frac{1}{5}\cos(5x) + C$

8) $\int \sec(2t)\tan(2t)dt$

$= \frac{1}{2}\sec(2t) + c$

9) $\int -3\csc x \cot x dx$

$= 3\csc x + c$

Differentiable Equations

Solve the *initial value* problem

$x=-1 \quad y=0$

1) $dx\left(\frac{dy}{dx}\right)=\left(9x^2-4x+5\right)dx \quad y(-1)=0 \quad (-1,0)$

$$\int dy=\int\left(9x^2-4x+5\right)dx$$

$$y+c_1=\frac{9}{3}x^3-\frac{4x^2}{2}+5x+c_2$$

$$y+c_1=3x^3-2x^2+5x+c_2$$

$-c_1 \qquad \frac{-c_1}{c_3}$

$$y=3x^3-2x^2+5x+c_3$$

$$y=3x^3-2x^2+5x+C$$

plug in the initial value $(-1,0)$

$$0=3(-1)^3-2(-1)^2+5(-1)+c$$

$$0=-3-2-5+c$$

$$0=-10+c$$

$$10=C$$

$$y=3x^3-2x^2+5x+10$$

2) $\frac{dy}{dx} = \frac{1}{x^2} + x \quad \overset{x}{}\overset{y}{} \; y(2) = 1$

$dy = \left(\frac{1}{x^2} + x\right) dx$

x^{-2}

$\int dy = \int \left(\frac{1}{x^2} + x\right) dx$

$y = \frac{x^{-1}}{-1} + \frac{x^2}{2} + C$

$y = -\frac{1}{x} + \frac{x^2}{2} + C$

initial value $(2, 1)$

$1 = -\frac{1}{(2)} + \frac{(2)^2}{2} + C$

$1 = -\frac{1}{2} + 2 + C$

$1 = 1\frac{1}{2} + C$

$-\frac{1}{2} = C$

$y = -\frac{1}{x} + \frac{x^2}{2} - \frac{1}{2}$

3) $dx^2 \dfrac{d^2y}{dx^2} = (2-6x)\,dx^2$ $y(0)=1$ $y'(0)=4$

$\int d^2y = \int (2-6x)\,dx^2$

$dy = 2x - 3x^2 + C$ $\rightarrow dx\left(\dfrac{dy}{dx}\right) = (2x - 3x^2 + 4)\,dx$

$\underline{y'(0)=4}$

$4 = 2(0) - 3(0)^2 + C$

$4 = C$

$\int dy = \int (2x - 3x^2 + 4)\,dx$

$y = x^2 - x^3 + 4x + C$

$\underline{y(0)=1}$

$1 = (0)^2 - (0)^3 + 4(0) + C$

$1 = C$

$\boxed{y = x^2 - x^3 + 4x + 1}$