# <u>Sections 6.3 and 6.4:</u> *(part 1)*
## Anti-derivatives

<u>Data comes in the following forms</u>

1. Tables (Numeric data)
2. Graph
3. Function

Integral = Area

<u>Three ways to integrate</u>

1. (1) RAM Rectangular Approx. Method } Typically used with tables
   (2) Trapezoid
2. Geometry (typically used with graphs)
3. Antiderivatives (typically used with functions)

<u>Review</u> *(basic anti-derivatives)*

$x^2 \Rightarrow$ $\frac{1}{3}x^3 + C$

$4x^3 \Rightarrow$ $x^4 + C$

$x^{\frac{1}{3}} \Rightarrow$ $\frac{3}{4}x^{\frac{4}{3}} + C$

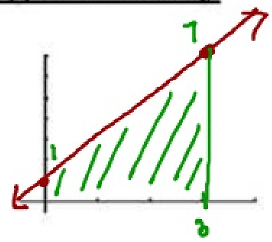
Using the anti-derivative method is referred to as the
Fundamental Theorem of Calculus (Part II)
*(Part I will come later)*

Evaluate each integral

1) $\int_0^3 (2x+1)dx$

*We will look at two possible methods for #1 only*

**Using Geometry**




$\int_0^3 (2x+1)dx$

$= \frac{1}{2}(3)(1+7)$

$= 12$

**OR**

**Using Anti-derivatives**

$\int_0^3 (2x+1)dx$

$= x^2 + x + C \Big|_0^3$

$= \left[(3)^2 + (3) + C\right] - \left[(0)^2 + (0) + C\right]$

$= 12 + C - C$

$= 12$

2) $\int_0^{\pi} \sin x \, dx$

$= -\cos x + C \Big|_0^{\pi}$

$= \left[-\cos(\pi) + C\right] - \left[-\cos(0) + C\right]$

$= 1 + C + 1 - C$

$= 2$

3) $\int_0^{2\pi} \sin x \, dx$

$= -\cos x \Big|_0^{2\pi}$

$= \left[-\cos(2\pi)\right] - \left[-\cos(0)\right]$

$= -1 + 1$

$= 0$

4) $\int_1^3 \left(\frac{1}{x} - \frac{1}{x^2}\right) dx$ $\nearrow x^{-2}$

$= \ln x + x^{-1} \Big|_1^3$

$= \left[\ln(3) + \frac{1}{3}\right] - \left[\ln(1) + \frac{1}{1}\right]$

$= \ln 3 + \frac{1}{3} - 0 - 1$

$= \ln 3 - \frac{2}{3}$

5) $\int_0^{\frac{5\pi}{3}} \cos x \, dx$

$= \sin x \Big|_0^{\frac{5\pi}{3}}$

$= \left[\sin\left(\frac{5\pi}{3}\right)\right] - \left[\sin(0)\right]$

$= -\frac{\sqrt{3}}{2} - 0$

$= \frac{-\sqrt{3}}{2}$

6) $\int_1^3 \left(e^x + x^{\frac{2}{3}}\right) dx$

$= e^x + \frac{3}{5} x^{\frac{5}{3}} \Big|_1^3$

$= \left[e^3 + \frac{3}{5}(3)^{\frac{5}{3}}\right] - \left[e^1 + \frac{3}{5}(1)^{\frac{5}{3}}\right]$

7) $\int_0^{\frac{\pi}{3}} \sec^2 x \, dx$

$= \tan x \Big|_0^{\pi/3}$

$= \left[\tan(\pi/3)\right] - \left[\tan(0)\right]$

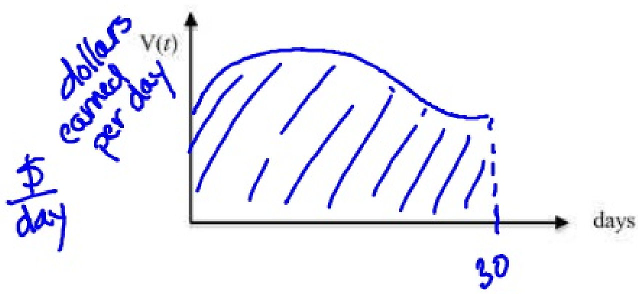
$= \sqrt{3} - 0$

$= \sqrt{3}$

## Average Value of a Function

$$\text{Average Value of a Function} = \frac{\int_a^b f(x)\,dx}{b-a} \quad \textbf{\textit{OR}} \quad \frac{1}{b-a}\int_a^b f(x)\,dx$$

**Example**




Note:

$$\int_0^{30} \underbrace{v(t)}_{\frac{\$}{day}}\,\underbrace{dt}_{\cdot\, day} = \$$$

v($t$) represents $ earned per day

$$\text{Average \$ per day} = \frac{\int_0^{30} v(t)\,dt \ \text{dollars}}{30-0 \ \text{days}} \quad \text{OR} \quad \frac{1}{30-0}\int_0^{30} v(t)\,dt$$

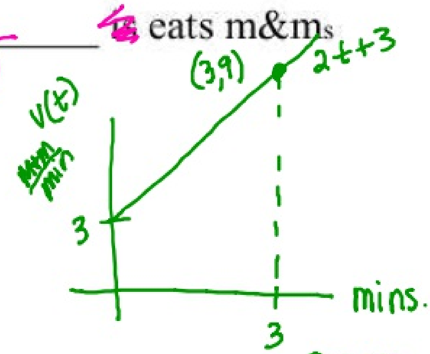
Answer will be $ per day

Write the formula if you were asked to find the *average* **$ per day** for days 3 thru 18?

$$\frac{1}{18-3}\int_3^{18} v(t)\,dt$$

8) v($t$) represents the rate at which Cody eats m&ms per minute.

$$v(t)=2t+3 \text{ on } [0, 3]$$




Determine the average value of v(t) and explain the meaning of the average.

Total M+M's eaten $= \int_0^3 v(t)\,dt$

@ t=0 rate $\frac{3\text{ m+m}}{\text{min}}$

@ t=3 rate $\frac{9\text{ mm}}{\text{min.}}$

Ave. value of v(t) on [0,3] $= \frac{1}{3-0}\int_0^3 (2t+3)\,dt$

$$= \frac{1}{3}\left[t^2+3t \Big|_0^3\right]$$

$$= \frac{1}{3}\left[(3^2+3(3))-((0)^2+3(0))\right]$$

$$= \frac{1}{3}[18]$$

$$= 6 \frac{\text{m+m}}{\text{min}}$$

time

*When* was Cody eating at the average rate?
(*When* = $t$-value)

$$\underset{2t+3}{\text{rate}} = \underset{6}{\text{ave. rate}}$$

$$2t = 3$$

$$t = \frac{3}{2} \text{ mins.}$$

At 1.5 mins. Cody was eating the m+m's at the average rate of 6 mm/min.

9) Find the average value of the function on the interval [0, 3]

$$y=(x-1)^2$$

$$=\frac{1}{3-0}\int_0^3 (x-1)^2\,dx$$

$$=\frac{1}{3}\int_0^3 \left(x^2-2x+1\right)dx$$

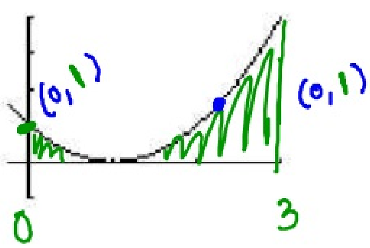
$$=\frac{1}{3}\left[\frac{x^3}{3}-x^2+x\Big|_0^3\right]$$

$$=\frac{1}{3}\left[\left(\frac{3^3}{3}-3^2+3\right)-\left(\frac{0^3}{3}-(0)^2+(0)\right)\right]=1$$

*When* does the function obtain the average value of the function?

(*When* = *x*-value) (*value of the function* = *y*-value)

$y=(x-1)^2$




$$(x-1)^2=1$$

$$\sqrt{(x-1)^2}=\pm\sqrt{1}$$

$$x-1=\pm 1$$

$$x=1\pm 1$$

$$\boxed{x=2,0}$$