Volumes of Solids

Day 1: Disc and Washer Method

Day 2: Shell Method

**Day 3: Cross Sections

<u>1st steps in all volume problems</u>

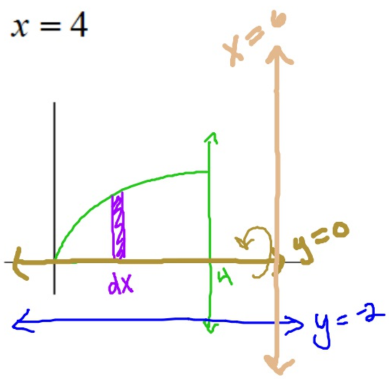
a) Draw curve
b) Draw representative rectangle
c) Determine ***dx*** or ***dy*** and set limits

Review

$y = \sqrt{x}$

$y = 0$

$x = 4$



rotate about $y = 0$ (x-axis) **evaluate *without* a calculator**

$$V = \pi\int_0^4 (\sqrt{x} - 0)^2\,dx$$

$$V = \pi\int_0^4 x\,dx$$

$$V = \pi\left(\frac{x^2}{2}\Big|_0^4\right)$$

$$V = \pi\left(\frac{4^2}{2} - \frac{0^2}{2}\right) = 8\pi$$

rotate about $y = -2$ ***set-up only***

$$V = \pi\int_0^4 \left[(\sqrt{x} - (-2))^2 - (0 - (-2))^2\right]dx$$

rotate about $x = 6$ ***set-up only***

$$V = 2\pi\int_0^4 (6 - x)(\sqrt{x} - 0)\,dx$$

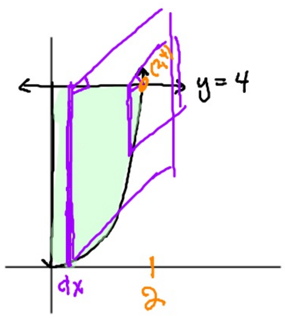
1) $y = x^2$

$y = 4$

$x = 0$

1st quadrant

square cross sections
perpendicular to the x-axis



Volumes using Cross Sections

$$V = \int_A^B (\text{Area of the cross section})\, dx \text{ or } dy$$

general formula

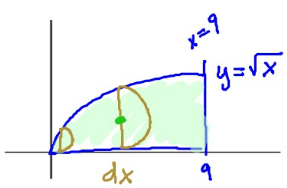
$$V = \int_0^2 (\text{ side })^2\, dx$$

$$V = \int_0^2 (4 - x^2)^2\, dx$$

2) $y = \sqrt{x}$

$x = 9$

$y = 0$



a) ***Semi-circle*** cross sections
perpendicular to the ***x-axis***

$\frac{\pi}{2} r^2$ = Area of semicircle

$$V = \frac{\pi}{2} \int_0^9 (\text{ radius })^2\, dx$$

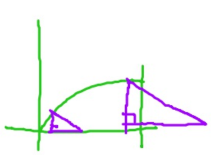
$$V = \frac{\pi}{2} \int_0^9 \left(\frac{1}{2}(\underbrace{\sqrt{x} - 0}_{\text{diameter}}) \right)^2 dx$$

b) ***Full circle*** cross sections
perpendicular to the ***x-axis***

πr^2 = Area of circle

to see a picture with full circles turn to page 410 #1

$$V = \pi \int_0^9 \left(\frac{1}{2}(\sqrt{x} - 0) \right)^2 dx$$



c) ***Right triangle*** cross sections
perpendicular to the ***x-axis***
with the ***height twice the base***

$\text{Area}_\triangle = \frac{1}{2} b h$

$$V = \frac{1}{2} \int_0^9 \underbrace{(\sqrt{x} - 0)}_{\text{base}} \underbrace{(2(\sqrt{x} - 0))}_{\text{height}}\, dx$$

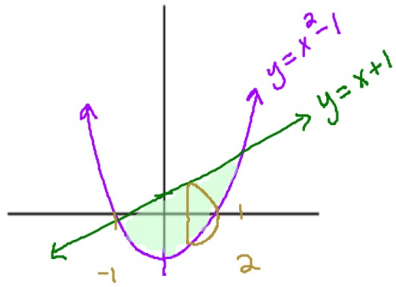
3) $y_1 = x + 1$

$y = x^2 - 1$

a) ***Semi-circle*** cross sections perpendicular to the ***x-axis***

Use your calculator to evaluate the volume

$$V = \frac{\pi}{2}\int_{-1}^{2}\left(\frac{1}{2}\left(\overset{y_1}{(x+1)} - \overset{y_2}{(x^2-1)}\right)\right)^2 dx$$



$x^2 - 1 = x + 1$

$x^2 - x - 2 = 0$

$(x-2)(x+1) = 0$

$x = 2, -1$

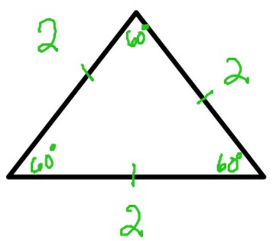
b) ***Equilateral triangle*** cross sections perpendicular to the ***x-axis***

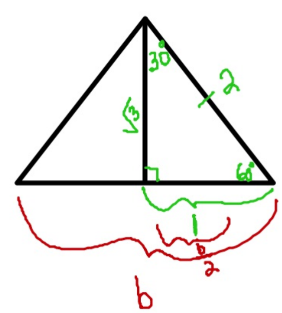
$$V = \frac{\sqrt{3}}{4}\int_{-1}^{2}(\text{base})^2\, dx$$

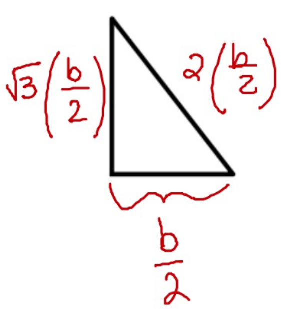
$$V = \frac{\sqrt{3}}{4}\int_{-1}^{2}\left((x+1) - (x^2-1)\right)^2 dx$$

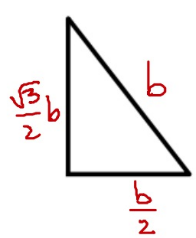
Note: You are expected to have the formula for the area of an equilateral triangle memorized for the Chapter 8 Test

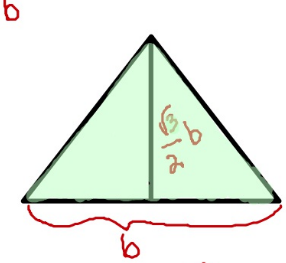
How to derive the formula for the area of an equilateral triangle











Area of △

$$\frac{1}{2}(b)\left(\frac{\sqrt{3}}{2}b\right) = \frac{\sqrt{3}}{4}b^2$$

$\frac{1}{2} \cdot \text{base} \cdot \text{height}$