Important things to know for the 3rd Quarter Final Exam

Read through each problem thoroughly

REVIEW ALL PREVIOUS END OF QUARTER INFORMATION SHEETS

Using calculators

nDeriv( ) and fnInt( ) nDeriv(y1, x, k) point; x ← graph the derivative

Find roots (zeros) and intersections

Def. of continuity
1) f(c) must exist
2) $\lim_{x \to c} f(x)$ exists
3) $f(c) = \lim_{x \to c} f(x)$

graphs on p.84 and p.85

Important Theorems

IVT: Intermediate Value Theorem
EVT: Extreme Value Theorem
MVT: Mean Value Theorem: $f'(c) = \frac{f(b)-f(a)}{b-a}$

Def. of derivative
$\lim_{h \to 0} \frac{f(x+h)-f(x)}{(x+h)-x}$

instantaneous vs. average rate of change

Position/Velocity/Acceleration

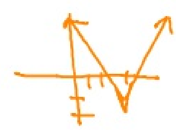
Velocity: speed and direction (determine where is v(t) + or -)
Speed = $|v(t)|$

Speed at t = 3: $|v(3)|$
Slowing down: v(3) and a(3) are different signs
Speeding up: v(3) and a(3) are the same signs

Rule for differentiability $\lim_{x \to a^-} f' = \lim_{x \to a^+} f'$

Determine the *greatest* speed by looking at the graph of velocity (*furthest distance* away from the *x*-axis)

p. 136 #9, 11



$f(x) = |x-3| - 2$
$f'(1) = -1$
$f'(3) = $ d.n.e.

Position: $p(t) = S(t)$
Velocity $p'(t) = v(t)$
Acceleration $p''(t) = v'(t) = a(t)$

Displacement = $\int_a^b v(t)dt \;=\; p(t)\Big|_a^b \;=\;$ p(b) − p(a) = Δ in position $\boxed{\text{Net Change}}$

Total distance = $\int_a^b |v(t)|dt$ $\boxed{\text{Gross Change}}$

$S(b) = S(a) + \int_a^b v(t)dt$

Position at time *b* : $p(b) \;=\; p(a) + \int_a^b v(t)dt$

Given a velocity (rate) function

Find the rate at $t = a$: plug *a* into the velocity function, find v(*a*) plug in

Find the total change from 0 to *a*: integrate the velocity function, evaluate $\int_0^a v(t)dt$ **OR** $\int_0^a |v(t)|dt$ v(t) is all above the x-axis

Find the total between *a* and *b*: integrate the velocity function, evaluate $\int_a^b v(t)dt$ **OR** $\int_a^b |v(t)|dt$

Fundamental Theorem of Calculus

Definite integral of the rate of change of a quantity over an interval interpreted as the change of the quantity over the interval: $\int_a^b f'(x)dx = f(b) - f(a)$ or $\int_a^b f(x)dx = F(b) - F(a)$ or $\int_a^b v(t)dt = p(b) - p(a)$

$$\frac{d}{dx}\int_a^x f(t)\,dt = f(x)$$

$$\frac{d}{dx}\int_a^{g(x)} f(t)\,dt = \frac{d}{dx}\left[F(t) \Big|_a^{g(x)} \right] = \frac{d}{dx}\left[F(g(x)) - F(a)\right] = f(g(x)) \cdot g'(x)$$

Data

Data is given in three different forms

- Function or Equation
- Chart → x y table
- Graph

Function or Equation

Identify the type of function given *(position, velocity or acceleration)*

1^{st} derivative of the function

    increasing and decreasing intervals
    relative and global - max / min

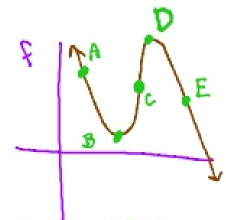
2^{nd} derivative of the function

    point(s) of inflection
    concave up ***or*** concave down

Drawing graph of the original function using the above information

Draw a graph of the 1^{st} derivative, given the original function

Draw a graph of the 2^{nd} derivative, given the original function or the 1^{st} derivative



A $f' < 0$ and $f'' > 0$

f is decr. f cc ↑

E $f' < 0$ and $f'' < 0$

Chart

t sec	$v(t)$ ft/sec

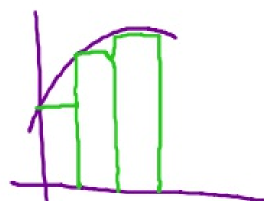
Identify the type of data given *(position, velocity, or acceleration)*

Graph the points from the given data values (label both axses)

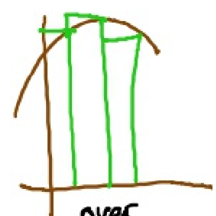
Find a derivative at a point or over an interval

Find an integral using an approximation method: RAM / Trapezoid

Explain the meaning of $\int$ or $\frac{dy}{dx}$



under approx.



over approx.

Average rate of change $= \frac{f(b) - f(a)}{b - a}$

Average value of a function $= \frac{1}{b-a}\int_a^b f(t)\,dt$

Justify answers in sentence form

Graph

Identify the type of graph given (position, velocity, or acceleration)

Fundamental Theorem of Calculus

Find the value of the function at any point

Find $\frac{dy}{dx}$ at any point or over an interval

Find $\int$ over an interval

Draw the graph of the function's derivative or integral

Symbolic $\frac{dy}{dx}$ or $\int$

Methods for Integrating
Straight-forward

Substitution

Rewrite

$$\frac{dP}{dt} = kP \Big\} \text{ directly proportional}$$

$$\int \frac{1}{P}\, dP = \int k\, dt$$

$$\ln|P| = kt + C$$

$$e^{kt+c} = P$$

$$P = Ce^{kt}$$

Exponential Growth and Decay
Law of Exponential Change: $\frac{dP}{dt} = kP$
Newton's Law of Cooling: $T = (T_0 - T_s)e^{kt} + T_s$

Slope fields Given a graph determine the function (mult. choice)

Graph $\frac{dy}{dx}$
Describe the field (notice patterns within the shown field)
Given an initial condition for the original function, draw the curve of the original function
Given an initial condition for the original function, solve for the original function
Look at the picture of $\frac{dy}{dx}$ and determine where "y" is non-differentiable and/or $\frac{dy}{dx} = 0$

(example: $\frac{dy}{dx} = \frac{y+1}{x}$)

Justify domain & range of the original function *(look at the graph of* $\frac{dy}{dx}$ *to find the undefined slopes)*

Units of measure --- do NOT forget the units of measure

position:	meters	gallons	liters	people	degrees	$^\circ F$	blah
velocity:	$\frac{meters}{hr}$	$\frac{gallons}{min}$	$\frac{liters}{sec}$	$\frac{people}{day}$	$\frac{degrees}{cm}$	$\frac{^\circ F}{mph}$	$\frac{blah}{dodo}$
acceleration:	$\frac{meters}{hr^2}$	$\frac{gallons}{min^2}$	$\frac{liters}{sec^2}$	$\frac{people}{day^2}$	$\frac{degrees}{cm^2}$	$\frac{^\circ F}{mph^2}$	$\frac{blah}{dodo^2}$

Related Rate

p. 255 # 11

Know

$$\frac{dV}{dt} = 100\pi \text{ ft}^3/\text{min}$$

$$V = \frac{4}{3}\pi r^3$$

Want

$$\frac{dr}{dt} = \quad \text{when } r=5$$

$$\frac{dV}{dt} = 4\pi r^2 \frac{dr}{dt}$$

$$100\pi = 4\pi (5)^2 \frac{dr}{dt}$$

$$\frac{100\pi}{4 \cdot 25\pi} = \frac{dr}{dt}$$

$$1 = \frac{dr}{dt}$$

1 ft/min

Area/Volume

Draw curves or enclosed region
Determine direction of representative rectangle
Find the Point(s) of Intersection
Determine the area of a region

$$Area = \int_A^B \qquad\qquad\qquad dx \text{ or } dy$$

Separate area - find the value of "*k*" that divides the region in half

Area under a curve p.287 #43

If $\int_2^5 f(x)dx = 18$,

then $\int_2^5 [f(x)+4]\,dx = ?$

$= \int_2^5 f(x)\,dx + \int_2^5 4\,dx$

$= 18 + 4x\Big|_2^5$

$= 18 + 4(5) - 4(2)$

$= 18 + 20 - 8$

$= 30$

OR

$= \int_2^5 f(x)dx + \int_2^5 4\,dx$

$= 18 + 4\underset{3}{\boxed{}}$

$= 18 + 12$

$= 30$

Volume

Revolve about the *x*-axis, *y*-axis, $x = \pm$ a, or $y = \pm$ a

Rectangle ***perpendicular*** to the axis of rotation

Disc(no gap): $V = \pi\int_A^B R^2\, dx$ or dy

Washer(gap): $V = \pi\int_A^B R^2 - r^2\, dx$ or dy

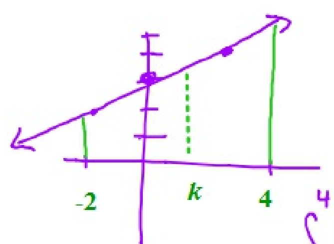
Rectangle ***parallel*** to the axis of rotation

Shell Method: $V = 2\pi\int_A^B (\qquad)(\qquad)\, dx$ or dy

Cross sections

$V = \int_A^B$ (AREA of the cross section) dx or dy

Given $y = \frac{x}{2} + 3$ on $[-2, 4]$ determine the value of k such that x = k divides the area in half.



$\int_{-2}^4 \left(\frac{x}{2} + 3\right) dx$

$= 21$

$\int_{-2}^k \left(\frac{x}{2} + 3\right) dx = \frac{21}{2} = 10.5$

$\frac{1}{4}x^2 + 3x\Big|_{-2}^k = 10.5$

$\left[\frac{1}{4}(k)^2 + 3(k)\right] - \left[\frac{1}{4}(-2)^2 + 3(-2)\right] = 10.5$

$\frac{1}{4}k^2 + 3k + 5 = 10.5$

$\frac{1}{4}k^2 + 3k - 5.5 = 0$

a = .25
b = 3
c = -5.5