Volumes of Solids

**Day 1: Disc and Washer Method

Day 2: Shell Method

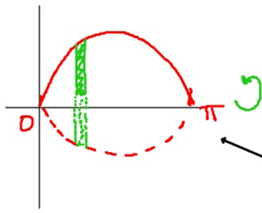
Day 3: Cross Sections

slice creates a cylinder

$V = \pi r^2 h$

$$\text{Volume} = \underbrace{\pi \int (\quad)^2 \, dx}_{\text{all cylinders added together}}$$

1) $y = \sin x \quad [0, \pi]$



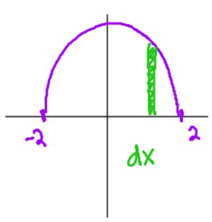
top − bottom

$$V = \pi \int_0^{\pi} (\sin x - 0)^2 \, dx$$

2) $y = 4 - x^2$

$y = 0$

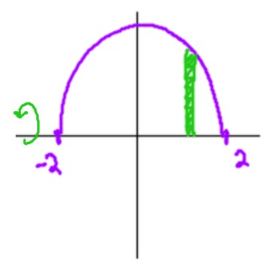
a) Draw curve
b) Draw representative rectangle
c) Determine ***dx*** or ***dy*** and set limits



rotate about *x-axis*

$$V = \pi \int_{-2}^{2} (\text{top} - \text{bottom})^2 \, dx$$

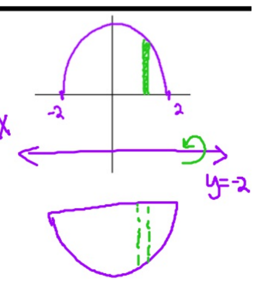
$$V = \pi \int_{-2}^{2} ((4-x^2) - 0)^2 \, dx$$



rotate about $y = -2$

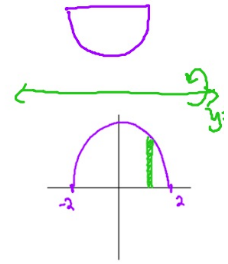
$$V = \pi \int_{-2}^{2} \left[(\quad)^2 - (\quad)^2 \right] dx \qquad R^2 - r^2$$

$$V = \pi \int_{-2}^{2} \left[(4-x^2-(-2))^2 - (0-(-2))^2 \right] dx$$



rotate about $y = 5$

$$V = \pi \int_{-2}^{2} \left[(5-0)^2 - (5-(4-x^2))^2 \right] dx \qquad R^2 - r^2$$

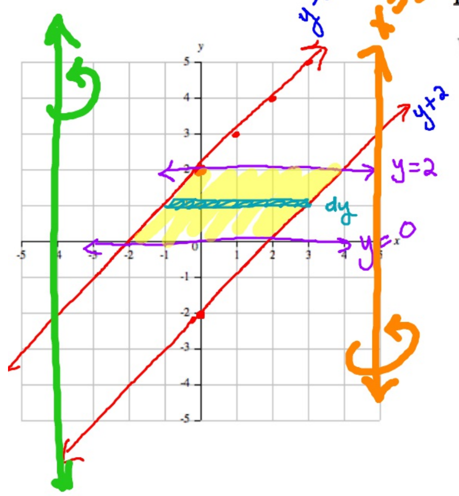


3) $y = x + 2$ $\quad X = y - 2 \leftarrow y_1$

$y = x - 2$ $\quad x = y + 2 \leftarrow y_2$

$y = 2$

$y = 0$



rotate about $x = 5$

$$V = \pi \int_0^2 \Big[\underbrace{(\qquad)^2}_{R^2} - \underbrace{(\qquad)^2}_{r^2}\Big] dy$$

$$V = \pi \int_0^2 \Big[\big(5 - \underbrace{(y-2)}_{y_1}\big)^2 - \big(5 - \underbrace{(y+2)}_{y_2}\big)^2\Big] dy = 64\pi$$

rotate about $x = -4$

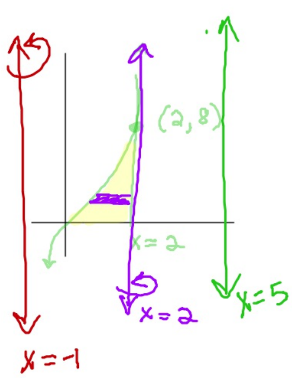
$$V = \pi \int_0^2 \Big[(\qquad)^2 - (\qquad)^2\Big] dy$$

$$V = \pi \int_0^2 \Big[\big((y+2)-(-4)\big)^2 - \big((y-2)-(-4)\big)^2\Big] dy$$

4) $y = x^3$ $\quad x = \sqrt[3]{y}$

$y = 0$

$x = 2$



rotate about $x = 2$

$$V = \pi \int_0^8 (\qquad)^2 \, dy$$

$$V = \pi \int_0^8 (2 - \sqrt[3]{y})^2 \, dy$$

rotate about $x = 5$

$$V = \pi \int_0^8 \Big[(\qquad)^2 - (\qquad)^2\Big] dy$$

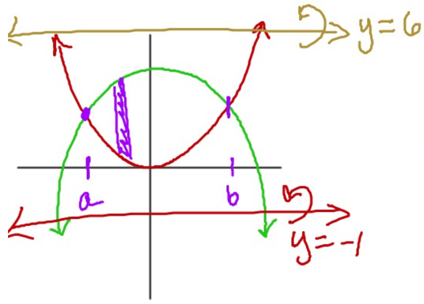
$$V = \pi \int_0^8 \Big[(5 - \sqrt[3]{y})^2 - (5 - 2)^2\Big] dy$$

rotate about $x = -1$

$$V = \pi \int_0^8 \Big[\big(2 - (-1)\big)^2 - \big(\sqrt[3]{y} - (-1)\big)^2\Big] dy$$

5) $y = x^2$

$y = 4 - x^2$



<u>rotate about $y = -1$</u>

$$V = \pi \int_a^b \left[\left((4-x^2)-(-1)\right)^2 - \left(x^2-(-1)\right)^2 \right] dx$$

<u>rotate about $y = 6$</u>

$$V = \pi \int_a^b \left[\left(6 - x^2\right)^2 - \left(6-(4-x^2)\right)^2 \right] dx$$