Volumes of Solids

Day 1: Disc and Washer Method Rectangle is $\perp$ to the axis of rotation

****Day 2: Shell Method** Rectangle is $\parallel$ to the axis of rotation

Day 3: Cross Sections

1st steps in all volume problems

a) Draw curve

b) Draw representative rectangle

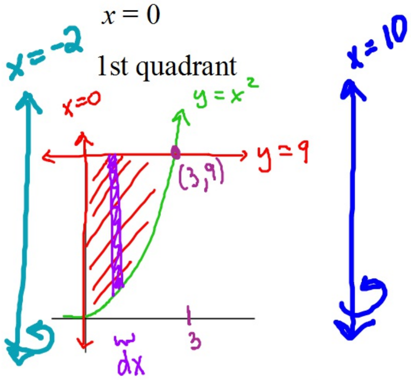
c) Determine ***dx*** or ***dy*** and set limits

1) $y = x^2$

$y = 9$

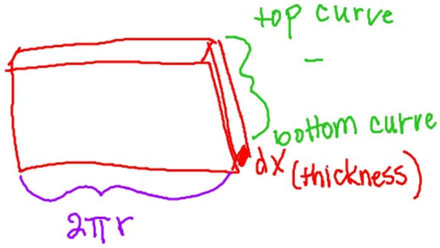
$x = 0$

1st quadrant



Area of rectangle

$2\pi(r)(\text{top} - \text{bottom})$



rotate about $x = 0$ (y-axis)

$$V = 2\pi\int_0^3 \underbrace{(x-0)}_{\text{radius}}\underbrace{(9-x^2)}_{\text{height of rectangle}}\,dx$$

rotate about $x = -2$

$$V = 2\pi\int_0^3 (x-(-2))(9-x^2)\,dx$$

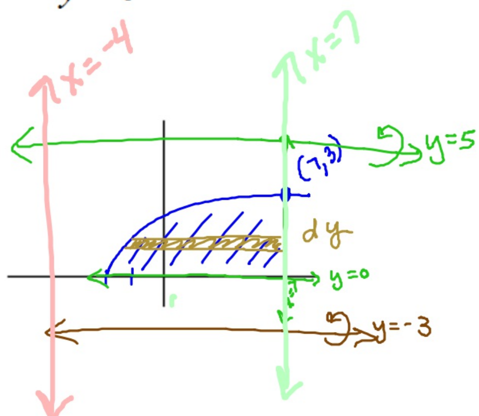
rotate about $x = 10$

$$V = 2\pi\int_0^3 (10-x)(9-x^2)\,dx$$

2) $y = \sqrt{x+2}$ $\quad y^2 = x+2$, $x = y^2 - 2$

$x = 7$

$y = 0$



rotate about $y = -3$

radius, rect. height

$$V = 2\pi \int_0^3 \left(y-(-3)\right)\left(7-(y^2-2)\right) dy$$

rotate about $y = 5$

$$V = 2\pi \int_0^3 (5-y)\left(7-(y^2-2)\right) dy$$

rotate about $x = 7$

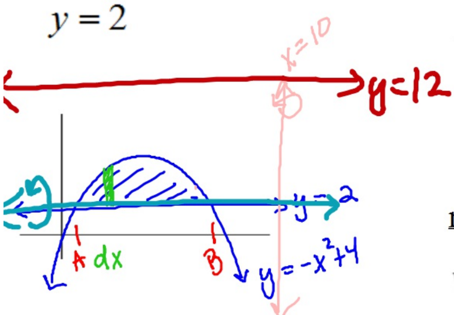
$$V = \pi \int_0^3 \left(7-(y^2-2)\right)^2 dy$$

rotate about $x = -4$

$$V = \pi \int_0^3 \left[\left(7-(-4)\right)^2 - \left(y^2-2-(-4)\right)^2\right] dy$$

3) $y = -x^2 + 4x$

$y = 2$



$-x^2+4x = 2$

$0 = x^2-4x+2$

$x = 0.585, 3.414$

$A \approx 0.585$

$B \approx 3.414$

rotate about y-axis

$$V = 2\pi \int_A^B (x-0)(-x^2+4x-2)\,dx$$

$V \approx 2\pi(7.54247) \approx 47.39075$

rotate about $x = 10$

$$V = 2\pi \int_A^B (10-x)(-x^2+4x-2)\,dx$$

rotate about $y = 2$

$$V = \pi \int_A^B (-x^2+4x-2)^2\,dx$$

rotate about $y = 12$

$$V = \pi \int_A^B \left[(12-2)^2 - \left(12-(-x^2+4x)\right)^2\right] dx$$