

# Integration by Parts

## Section 7.3

$$\int \underline{u} \underline{dv} = uv - \int v du$$

1.  $\int \underline{x} \underline{\cos x} dx$

$$\begin{array}{l} u = x \rightarrow v = \sin x \\ du = dx \rightarrow dv = \cos x dx \end{array}$$

$$\begin{aligned} &= x \cdot \sin x - \int \sin x dx \\ &= x \sin x - (-\cos x + C) \\ &= x \sin x + \cos x + C \end{aligned}$$

2.  $\int \underline{x} \underline{e^x} dx$

$$\begin{array}{l} u = x \rightarrow v = e^x \\ du = dx \rightarrow dv = e^x dx \end{array}$$

$$\begin{aligned} &= x e^x - \int e^x dx \\ &= x \cdot e^x - e^x + C \end{aligned}$$

3.  $\int \underline{x} \underline{\csc^2 x} dx$

$$\begin{array}{l} u = x \rightarrow v = -\cot x \\ du = dx \rightarrow dv = \csc^2 x dx \end{array}$$

$$\begin{aligned} &= x(-\cot x) - \int -\cot x dx \\ &= -x \cot x + \int \frac{\cos x}{\sin x} dx \\ &\quad m = \sin x \\ &\quad dm = \cos x dx \\ &= -x \cot x + \int \frac{1}{m} dm \\ &= -x \cot x + \ln|m| + C \\ &= -x \cot x + \ln|\sin x| + C \end{aligned}$$

4.  $\int \ln x dx$

$$\begin{array}{l} u = \ln x \rightarrow v = x \\ du = \frac{1}{x} dx \rightarrow dv = dx \end{array}$$

$$\begin{aligned} &= \ln x \cdot x - \int x \cdot \frac{1}{x} dx \\ &= x \ln x - \int dx \\ &= x \ln x - x + C \end{aligned}$$

## Tabular Method

5.  $\int x^3 \sin x \, dx$

$\frac{dx}{x^3}$	$\int$	
$3x^2$	$\sin x$	
$6x$	$-\cos x$	+
$6$	$-\sin x$	-
$0$	$\cos x$	+
	$\sin x$	-

$$= -x^3 \cos x + 3x^2 \sin x + 6x \cos x - 6 \sin x + C$$

L agr.  
 I nverse  
 P olyn  
 E xponent  
 T rig

can't use tabular  
 If one of the expressions is a polynomial  
 (exception  
 $e^{ax}$  ok  
 $e^{x^2}$  not ok)

6.  $\int x^2 e^{5x} \, dx$

$\frac{dx}{x^2}$	$\int$	
$2x$	$e^{5x}$	
$2$	$\frac{1}{5} e^{5x}$	+
$0$	$\frac{1}{25} e^{5x}$	-
	$\frac{1}{125} e^{5x}$	+

$$= \frac{1}{5} x^2 e^{5x} - \frac{2x}{25} e^{5x} + \frac{2}{125} e^{5x} + C$$

7.  $\int_1^4 x^5 \ln x \, dx$

$$\begin{aligned}
 u &= \ln x \rightarrow v = \frac{1}{6} x^6 \\
 du &= \frac{1}{x} dx \rightarrow dv = x^5 dx \\
 &= \ln x \cdot \frac{1}{6} x^6 - \int_1^4 \frac{1}{6} x^6 \cdot \frac{1}{x} dx \\
 &= \frac{x^6}{6} \ln x - \frac{1}{6} \int_1^4 x^5 dx \\
 &= \frac{x^6}{6} \ln x - \frac{1}{6} \frac{x^6}{6} \Big|_1^4 \\
 &= \frac{x^6}{6} \ln x - \frac{x^6}{36} \Big|_1^4 \\
 &= \frac{(4)^6}{6} \ln(4) - \frac{(4)^6}{36} - \left( \frac{(1)^6}{6} \ln(1) - \frac{(1)^6}{36} \right)
 \end{aligned}$$