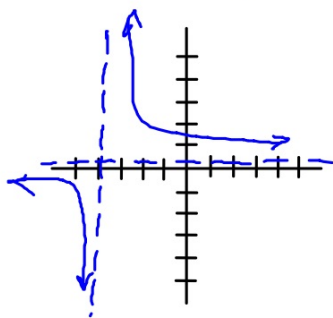


Limits Involving $\pm\infty$
Section 2.2

Review:

$$f(x) = \frac{1}{x+4}$$



$$\lim_{x \rightarrow -4^-} f(x) = -\infty$$

$$\lim_{x \rightarrow -4^+} f(x) = \infty$$

$$\lim_{x \rightarrow \infty} f(x) = 0$$

$$\lim_{x \rightarrow -\infty} f(x) = 0$$

End Behavior Models

$$f(x) = \frac{3x^2 + 2x - 1}{4x + 3}$$

$$\text{E.B.M. } y = \frac{3}{4}x$$

$$f(x) = \frac{14x^3 + 2x^3 - 1}{7x^3 - 2x^2}$$

$$\text{E.B.M. } y = 2x^2$$

$$f(x) = \frac{-6x^2 + 3x - 1}{5x + x^2}$$

$$\text{E.B.M. } y = -6$$

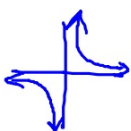
Definition for Horizontal Asymptote:

H.A for $f(x)$ is $y=a$ and/or $y=b$

if $\lim_{x \rightarrow \infty} f(x) = a$ and $\lim_{x \rightarrow -\infty} f(x) = b$

Find the $\lim_{x \rightarrow \infty} f(x)$, $\lim_{x \rightarrow -\infty} f(x)$, and identify the horizontal asymptote.


1. $f(x) = \frac{x+3}{x^2-10x-24}$

E.B.M. $y = \frac{1}{x}$ 

$\lim_{x \rightarrow \infty} f(x) = 0$
 $\lim_{x \rightarrow -\infty} f(x) = 0$

H.A.
 $y = 0$

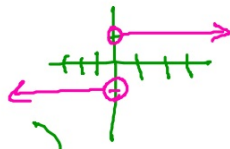
2. $f(x) = \frac{-x^3-3x^3+2x^2+4}{1-2x+x^2}$

E.B.M. $y = -x^3$ 

$\lim_{x \rightarrow \infty} f(x) = -\infty$
 $= \text{d.n.e.}$
 $\lim_{x \rightarrow -\infty} f(x) = \infty$
 $= \text{d.n.e.}$

H.A.
d.n.e.

3. $f(x) = \frac{|x|}{x}$

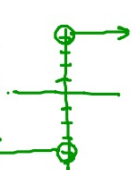


$$\lim_{x \rightarrow \infty} \frac{|x|}{x} = 1$$

$$\lim_{x \rightarrow -\infty} \frac{|x|}{x} = -1$$

H.A. $y=1, y=-1$

4. $f(x) = \frac{4|x| - 2}{x + 1}$ E.B.M.

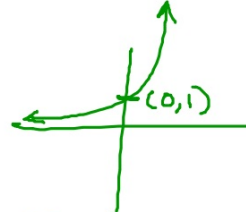


$$\lim_{x \rightarrow \infty} f(x) = 4$$

$$\lim_{x \rightarrow -\infty} f(x) = -4$$

H.A. $y=4, y=-4$

5. $f(x) = e^x$

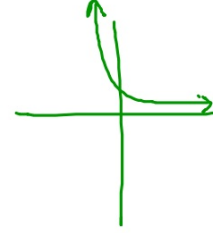


$$\lim_{x \rightarrow \infty} e^x = \infty$$

$$\lim_{x \rightarrow -\infty} e^x = 0$$

H.A. $y=0$

6. $f(x) = e^{-x}$



$$\lim_{x \rightarrow \infty} e^{-x} = 0$$

$$\lim_{x \rightarrow -\infty} e^{-x} = \infty$$

H.A. $y=0$

7. $f(x) = x^2 + e^x$

$$\lim_{x \rightarrow \infty} (x^2 + e^x) \quad \left. \begin{array}{l} \downarrow \quad \downarrow \\ \infty + \infty = \infty \\ \downarrow \quad \downarrow \\ \infty + 0 = \infty \end{array} \right\} \begin{array}{l} \text{H.A.} \\ \text{d.n.e.} \end{array}$$

8. $f(x) = x^2 \cdot e^x$

$$\lim_{x \rightarrow \infty} (x^2 \cdot e^x) \quad \left. \begin{array}{l} \downarrow \quad \downarrow \\ \infty \cdot \infty = \infty \\ \downarrow \quad \downarrow \\ \infty \cdot 0 = 0 \end{array} \right\} \begin{array}{l} \text{H.A.} \\ y=0 \end{array}$$

9. $f(x) = \frac{\cos x}{x}$

$$\lim_{x \rightarrow \infty} \frac{\cos x}{x} \quad \left. \begin{array}{l} \text{small} \# \\ \text{HUGE} \\ y=0 \\ \text{small} \# \\ \text{HUGE} \\ y=0 \end{array} \right\} \begin{array}{l} \text{H.A.} \\ y=0 \end{array}$$

10. $f(x) = \frac{\sin(\frac{1}{x})}{\frac{1}{x} + 2}$

$$\lim_{x \rightarrow \infty} f(x) = \frac{0}{0+2} = \frac{0}{2} = 0$$

$$\lim_{x \rightarrow -\infty} f(x) = 0$$