Name: ____________________

Period: _____

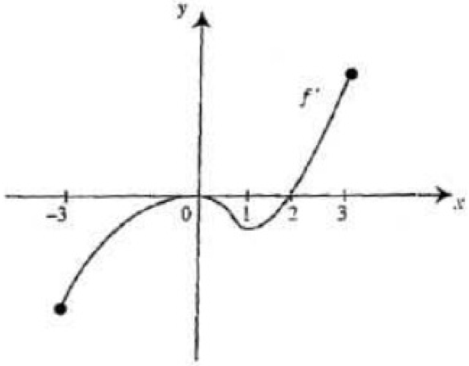
# Chart Problems

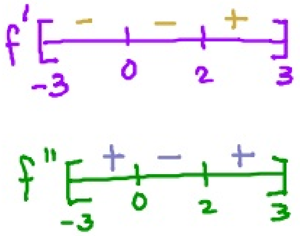
## Chapter 5

**1) The graph of $f'$ on $[-3,3]$ is shown. Find the values of $x$ on $[-3,3]$ such that**

**(a) $f$ is increasing** f is inc. on (2,3) b/c f'>0 there

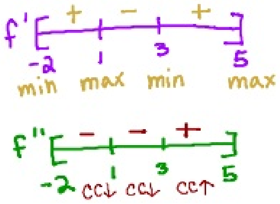
**(b) $f$ is concave downward.** f is cc↓ on (0,1) b/c f''<0 there





**2) A function $f$ is continuous on the interval $[-2,5]$ with $f(-2) = 10$ and $f(5) = 6$ and the following properties:**

| Intervals | (-2,1) | $x = 1$ | (1,3) | $x = 3$ | (3,5) |
|---|---|---|---|---|---|
| $f'$ | + | 0 | - | undefined | + |
| $f''$ | - | 0 | - | undefined | + |



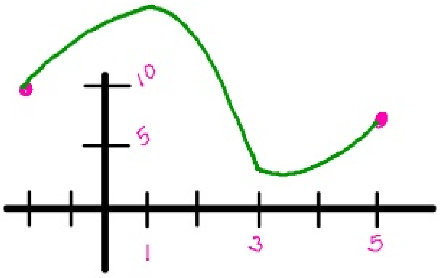
(a) Find the intervals on which $f$ is increasing (-2,1) (3,5) b/c f'>0
Find the intervals on which $f$ is decreasing. (1,3) b/c f'<0

(b) Find the $x$-value(s) where $f$ has its extrema. f has max @ X=1,5
f has min. @ X=-2,3

(c) Find the $x$-value(s) where $f$ has points of inflection. f has a pt. of infl. @ X=3

(d) Find the intervals on where $f$ is concave is upward. f is cc↑ (3,5)
Find the intervals on where $f$ is concave is downward. f is cc↓ (-2,1) (1,3)

(e) Sketch a possible graph of $f$.



must use points given
f(-2)=10
f(5)=6

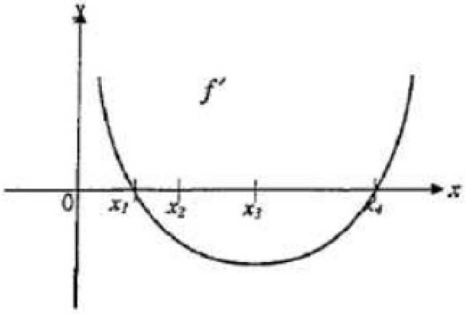
**3) Given the graph of $f'$, determine at which of the four values of $x$ $(x_1, x_2, x_3, x_4)$ does $f$ have**

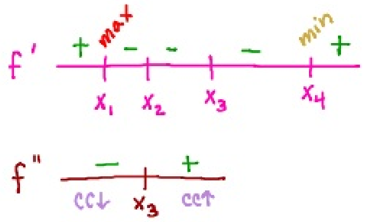
(a) the largest value, $x_1$

(b) the smallest value, $x_4$

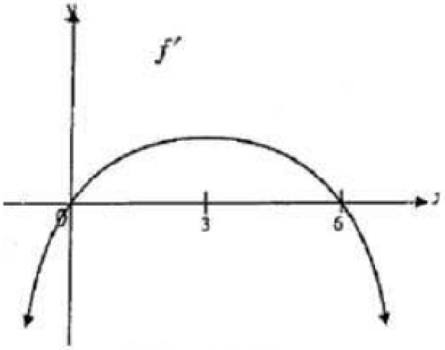
(c) a point of inflection, $x_3$

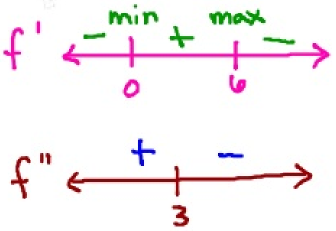
(d) and at which of the four values of $x$ does $f''$ have the largest value. $x_4$





**4) Given the graph of $f'$, find where the function $f$**





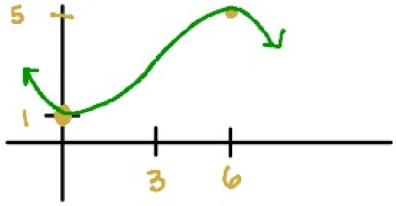
(a) has its relative extrema at $x =$ 6 max.

(b) is increasing or decreasing, inc. $(0,6)$
decr. $(-\infty,0)$ $(6,\infty)$

(c) has its point(s) of inflection, pt. of infl. $x=3$

(d) is concave upward or downward, CC↑ $(-\infty,3)$
CC↓ $(3,\infty)$

(e) Given: $f(0) = 1$ and $f(6) = 5$, draw a sketch of $f$.



**5) Determine the intervals in which the graph of $f(x)=\frac{x^2+9}{x^2-25}$ is concave upward or downward.**

→ Domain: ℝ ex. $x \neq -5, 5$

$$f'=\frac{(x^2-25)(2x)-(x^2+9)(2x)}{(x^2-25)^2}$$

$$f'=\frac{2x^3-50x-2x^3-18x}{(x^2-25)^2}$$

$$f'=\frac{-68x}{(x^2-25)^2}$$

$$f''=\frac{(x^2-25)^2(-68)-(-68x)(2(x^2-25)^1(2x))}{[(x^2-25)^2]^2}$$

$$f''=\frac{-68(x^2-25)^2+272x^2(x^2-25)}{(x^2-25)^4}$$

$$f''=\frac{(x^2-25)(-68(x^2-25)+272x^2)}{(x^2-25)^4}$$

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**5) Determine the intervals in which the graph of $f(x)=\frac{x^2+9}{x^2-25}$ is concave upward or downward.** ⟶ Domain: ℝ ex. $x \neq -5, 5$

$$f' = \frac{(x^2-25)(2x)-(x^2+9)(2x)}{(x^2-25)^2}$$

$$f' = \frac{2x^3-50x-2x^3-18x}{(x^2-25)^2}$$

$$f' = \frac{-68x}{(x^2-25)^2}$$

$$f'' = \frac{(x^2-25)^2(-68)-(-68x)(2(x^2-25)^1(2x))}{[(x^2-25)^2]^2}$$

$$f'' = \frac{68(x^2-25)^2+272x^2(x^2-25)}{(x^2-25)^4}$$

$$f'' = \frac{(x^2-25)(-68(x^2-25)+272x^2)}{(x^2-25)^4}$$

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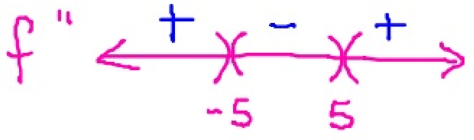
$\underline{f''=0}$ or $\underline{f''=\text{undef.}}$

$x = 5, -5$

$$(x^2-25)(-68x^2+1700+272x^2)=0$$

$$\underbrace{(x^2-25)}_{x=\pm 5}\underbrace{(204x^2+1700)}_{\text{never equals zero}}=0$$



∴ f is cc↑ $(-\infty,-5)(5,\infty)$

cc↓ $(-5,5)$

**6) Given $f(x) = \sqrt{(25-x^2)}$ on $[-5,5]$**

Show that the *absolute minimum is 0* and the *absolute maximum is 5*.

$f'(x) = \frac{1}{2}(25-x^2)^{-1/2}(-2x)$ $\quad \frac{f'=0}{x=0} \quad \frac{f'=\text{undef.}}{x=\pm 5}$

$f'(x) = \frac{-x}{\sqrt{25-x^2}}$

$f'$ min [ + max − ] min, at $-5$, $0$, $5$

Since $f(5)=f(-5)$ they are both abs. min.

**7) Given $f(x) = x + \sin x \quad 0 \le x \le 2\pi$,** find any points of inflection of $f$.

$f' = 1+\cos x$ $\quad \underline{f''=0}$ $\quad \frac{f''=\text{undef}}{\text{never}}$

$f'' = -\sin x$ $\quad -\sin x = 0$

$\sin x = 0$

$x = 0, \pi, 2\pi$

$f''$ [ − | + ] at $0$, $\pi$, $2\pi$ $\quad$ p.o.i. $(\pi, \pi)$

**8) Evaluate** $\lim\limits_{x\to 100} \frac{x-100}{\sqrt{x}-10}$ $= \lim\limits_{x\to 100} \frac{\cancel{(\sqrt{x}-10)}(\sqrt{x}+10)}{\cancel{\sqrt{x}-10}} = \lim\limits_{x\to 100} \sqrt{x}+10 = \sqrt{100}+10 = 20$

**9) If the position function of a particle is** $s(t) = \frac{t^3}{3} - 3t^2 + 4$,

find the velocity and position of the particle when its acceleration is 0.

$s' = v = t^2 - 6t$ $\quad a(t) = 0$

$s'' = v' = a = 2t - 6$ $\quad 2t - 6 = 0$

$t = 3$

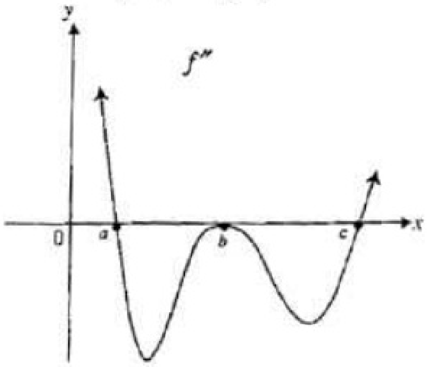
position $s(3) = \frac{(3)^3}{3} - 3(3)^2 + 4$

$s(3) = -14$

velocity $v(3) = (3)^2 - 6(3)$

$v(3) = -9$

**10) Given the graph of $f''$, determine the values of $x$ which the function $f$ has a point of inflection.**



$f''$ ← + | − | − | + → at $a$, $b$, $c$

f has points of inflection @ $x = a, c$

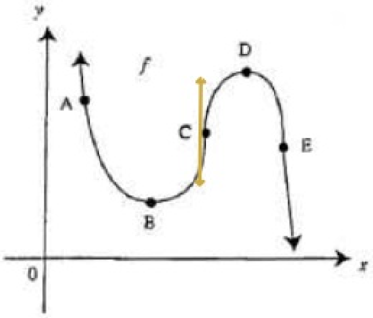
**11) Given the graph of the function $f$, identity the points where:**

a) $f' < 0$ and $f'' > 0$ A

b) $f' < 0$ and $f'' < 0$ E

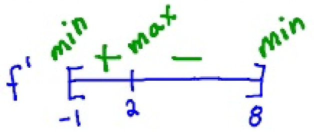
c) $f' = 0$ B, D

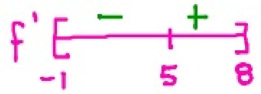
d) $f''$ does not exist C



**12) A function *f* is continuous on the interval $[-1,8]$ with $f(0)=0$, $f(2)=3$, and $f(8) = \frac{1}{2}$ and the following properties:**

| Interv | (-1,2) | $x = 2$ | (2,5) | $x = 5$ | (5,8) |
|---|---|---|---|---|---|
| $f'$ | + | 0 | - | - | - |
| $f''$ | - | - | - | 0 | + |





(a) Find the intervals on which $f$ is increasing (-1,2)

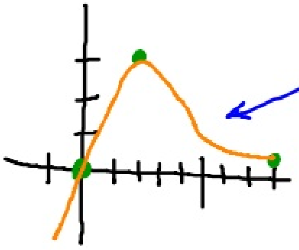
Find the intervals on which $f$ is decreasing. (2,8)

(b) Find the $x$-value(s) where $f$ has its extrema. min @ x= -1, 8
max @ x = 2

(c) Find the $x$-value(s) where $f$ has points of inflection. X = 5

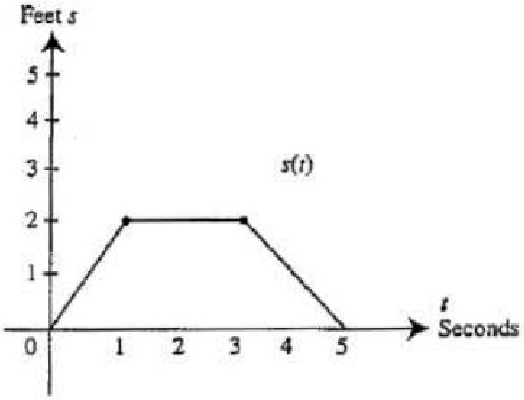
(d) Find the intervals on where $f$ is concave is upward. CC↑ (5,8)
Find the intervals on where $f$ is concave is downward. CC↓ (-1,5)

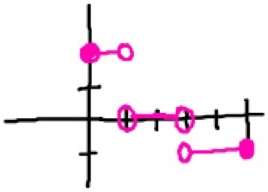
(e) Sketch a possible graph of $f$.



This is a poor attempt to have a point of inflection at x=5

**13) The graph represents the distance in feet covered by a moving particle in $t$ seconds. Draw a sketch of the corresponding velocity function.**





**14) A ball dropped from the top of a 640-foot building.**
**The position function of the ball is $s(t) = -16t^2 + 640$ where $t$ is measured in seconds and $s(t)$ is in feet.**

notice

$s' = v = -32t$

**Find:**

(a) The position of the ball after 4 seconds. $S(4) = -16(4)^2 + 640 = 384$ ft.

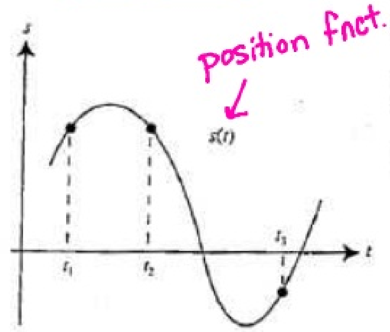
(b) The instantaneous velocity of the ball at $t = 4$. $V(4) = -32(4) = -128$ ft/sec.

(c) The average velocity for the first 4 seconds. $\frac{S(4)-S(0)}{4-0} = \frac{384-640}{4} = -64$ ft/sec.

(d) When will the ball hit the ground? $S(t) = 0$, $-16t^2 + 640 = 0$, $t^2 = \frac{640}{16}$, $t = \sqrt{40}$ sec.

(e) Determine the speed of the ball when it hits the ground. $\left|V\left(\frac{640}{16}\right)\right| = \left|-32\sqrt{40}\right| = 32\sqrt{40}$ ft/sec

**15) The position function of a moving particle is shown in the graph. For which value(s) of t ($t_1$, $t_2$, $t_3$) is:**



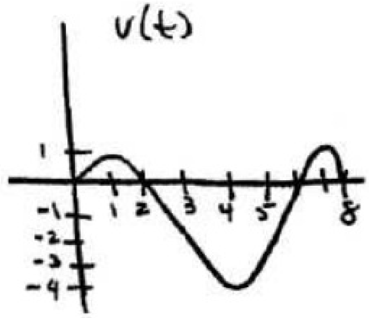
(a) The particle moving to the left? $t_2$ b/c $s' = v < 0$

(b) The acceleration negative? $t_1$ and $t_2$ b/c $s'' = a < 0$

(c) The particle moving to the right and slowing down?
$t_1$ b/c $s' = v > 0$ and $s'' = a < 0$

(Slowing down
v and a are opposite signs)

**16) The velocity function of a particle is shown in the graph.**



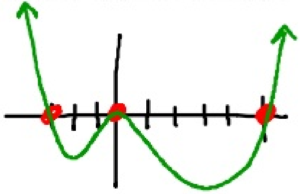
(a) When does the particle reverse direction? $t = 2, 6$

(b) When is the acceleration 0? $t = 1, 4, 7$

(c) When is the speed the greatest? $t = 4$

(d) When is the particle moving left/right? left (2,6)
right (0,2)(6,8)

**17) If $f''(x)=x^2(x+3)(x-5)$, find the values of $x$ at which the graph of $f$ has a change of concavity.**



$x=-3,5$

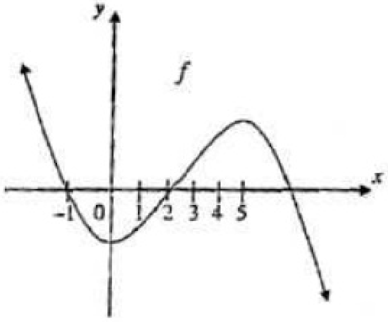
**19) Given the graph of $f$ at which values of $x$ is:**

(a) $f'(x)=0$ $x=0,5$

(b) $f''(x)=0$ $x=2.5$ (or $x=3$)

(c) $f'$ is a decreasing function

$(-\infty,0)(5,\infty)$



**20. $f(x)=xe^x$ Find the values of $x$ such that**

(a) $f$ is increasing $(-1,\infty)$

(b) $f$ is cc↓ $(-\infty,-2)$

$f'(x)=xe^x+e^x$

$f'(x)=e^x(x+1)$

$\underline{f'=0}$ $\underline{f'=\text{undef.}}$

$\underbrace{e^x}_{\substack{\text{always}\\ +}}\underbrace{(x+1)}_{x=-1}=0$ never

$f'\ (\overset{-}{\quad}\underset{-1}{|}\overset{+}{\quad})$

$f''(x)=xe^x+e^x+e^x$

$f'(x)=xe^x+2e^x$

$\underline{f''=0}$ $\underline{f''=\text{undef.}}$

$\underbrace{e^x}_{\substack{\text{always}\\ +}}\underbrace{(x+2)}_{x=-2}=0$ never

$f''\ (\overset{-}{\quad}\underset{-2}{|}\overset{+}{\quad})$