Discovering the Mean Value Theorem

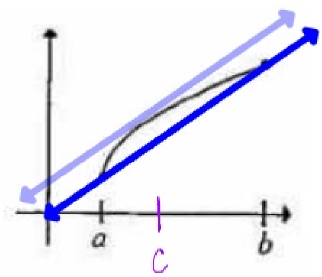
For examples 1 – 7 draw these lines *(only if possible)*

If $y = f(x)$ **is continuous** at every point of the closed interval **[a, b]** and **differentiable** at every point of its interior **(a, b)**,

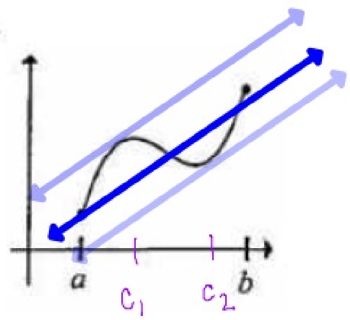
(a) Draw the secant line between the two points (a, f(a)) and (b, f(b))

(b) Draw all tangent lines parallel to the secant line.

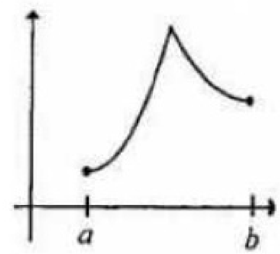
1.



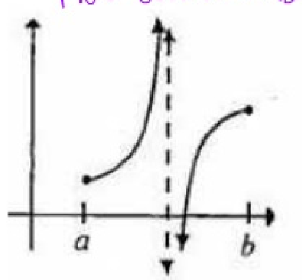
2.



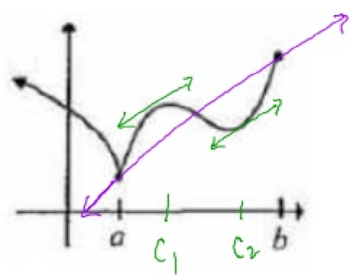
3. not diff



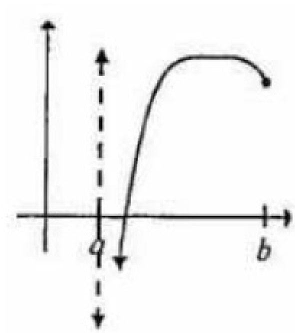
4. not continuous



5.

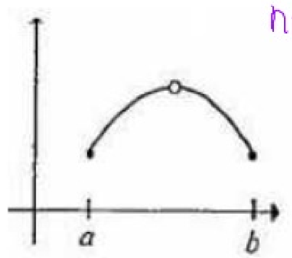


6. not a closed interval



7.

not continuous



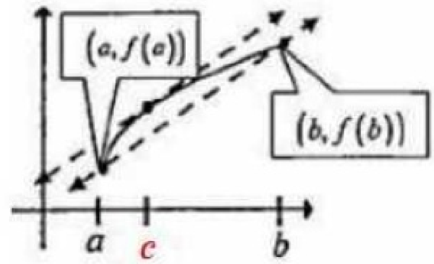
MEAN VALUE THEOREM

If f is continuous on [a, b] and differentiable on (a, b), then there is a number c in (a, b) such that

$$f'(c) = \frac{f(b)-f(a)}{b-a}$$

tangent slope
(instantaneous rate of change)

secant slope
(average rate of change)



Write in your own words . . .

Find the value of "c" that satisfies the M.V.T.

1. $f(x) = x^2 - x - 6$ on [0, 3]

$\frac{f(3)-f(0)}{3-0} = f'(c)$

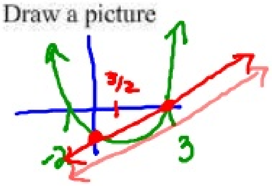
$\frac{\overbrace{[(3)^2-(3)-6]}^{0}-\overbrace{[(0)^2-(0)-6]}^{-6}}{3} = 2c-1$

$\frac{6}{3} = 2c-1$

$2 = 2c-1$

$\frac{3}{2} = c$

Draw a picture



2. $p(x) = \ln(x+2)$ on [-1, 3]

$\frac{p(3)-p(-1)}{3-(-1)} = p'(c)$

$\frac{\ln(3+2)-\ln(-1+2)}{4} = \frac{1}{c+2}$

$\frac{\ln(5)-\ln(1)}{4} = \frac{1}{c+2}$

$(c+2)\left(\frac{\cancel{4}}{\cancel{\ln 5}}\right)\frac{\cancel{\ln(5)}}{\cancel{4}} = \frac{1}{\cancel{c+2}}\left(\cancel{c+2}\right)\left(\frac{4}{\ln 5}\right)$

$c+2 = \frac{4}{\ln 5}$

$c = \frac{4}{\ln 5} - 2$

3. $f(x) = 3 - \frac{6}{x}$ on [3, 6] $\rightarrow f(x) = 3-6x^{-1}$

$\frac{f(6)-f(3)}{6-3} = 6c^{-2}$

$\frac{[3-1]-[3-2]}{3} = \frac{6}{c^2}$

$\frac{2-1}{3} = \frac{6}{c^2}$

$\frac{1}{3} = \frac{6}{c^2}$

$c^2 = 18$

$c = \pm\sqrt{18}$

$c = \pm 3\sqrt{2}$

$c = 3\sqrt{2}$ only

Mean Value Theorem Assignment

Determine if the Mean Value Theorem applies on the given interval.

(a) If not, explain why *or* (b) If M.V.T. applies, find the value of c that satisfies the M.V.T.

1. $f(x)=|x|$ on [-1, 3]

2. $f(x)=x^2-2x$ on [1, 3]

3. $f(x)=x^2-3x+2$ on [1, 2]

4. $f(x)=x^{\frac{2}{3}}$ on [-2, 2]

5. $f(x)=x^{\frac{2}{3}}$ on [0, 1]

6. $f(x)=\dfrac{1}{x-4}$ on [2, 6]

7. $f(x)=\dfrac{x^2-x}{x}$ on [-1, 1]

8. $f(x)=x^2-2x$ on [0, 2]

Find the c-value guaranteed by the Mean Value Theorem for the given function on the given interval. You may use a calculator.

9. $f(x)=x^3-2x^2+1$ on [-1, 2]

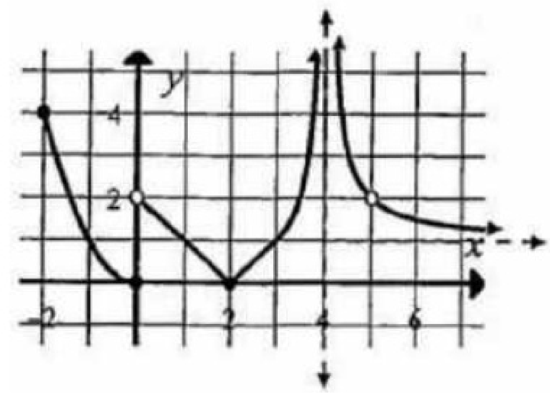
10. $f(x)=\dfrac{1}{x-1}$ on [2, 3]

11. The height, in feet, of an object a time t seconds is given by $h(t)=-16t^2+200$

a. Find the average velocity of the object during the first 3 seconds.

b. Use the Mean Value Theorem to find the time at which the object's instantaneous velocity equals the average velocity.

12. Use the graph of at the right for these problems.



a) Find $\lim_{x \to 0^+} f(x)$

b) Find $\lim_{x \to 0^-} f(x)$

c) Find $\lim_{x \to 4} f(x)$

d) Find $\lim_{x \to \infty} f(x)$

e) Find $\lim_{x \to 3} f(x)$

f) List the discontinuities of f.

g) Which of these discontinuities are removable?

h) Find the absolute maximum of $f(x)$ on [-2, 3]

i) Find the absolute maximum of $f(x)$ on [-2, 3]

j) Find $f'(1)$

k) Find $f''(1)$

l) List all x-values where $f'(x)$ does not exist.

m) List all x-values where $f(x)$ has a local minimum.

n) List all x-values where $f(x)$ has a local maximum.

Find all possible functions "f" with the given derivative

13. $f'(x) = x$

14. $f'(x) = 2$

15. $f'(x) = 3x^2 - 2x + 1$

16. $f'(x) = \sin x$

17. $f'(x) = e^x$

18. $f'(x) = \dfrac{1}{x-1}$

19. $f'(x) = \sec x \tan x$

20. $f'(x) = 5\cos(5x)$

21. $f'(x) = -3e^{-3x}$