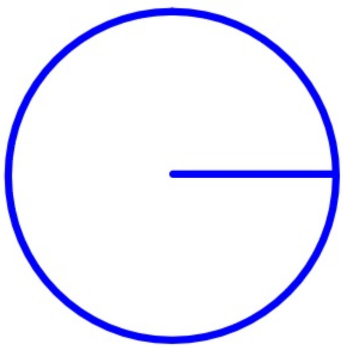
<u>Note:</u> Formulas are on p. 572 in your textbook

- Read the problem
- Draw a picture
- Write formulas for ***<u>given</u>*** and ***<u>optimization</u>***
- **<u>Solve</u>** for the **<u>variable</u>** *in the given* to use in the *optimization*
- **<u>Substitute the variable</u>** into the optimization formula
- *Write the equation in terms of* ***<u>one variable</u>***
- Simplify *completely !*
- Differentiate
- Find critical point(s)
- Use what you have found to find the other variable
- Answer the question

Notes for Sec. 5.4: Modeling and Optimization

Formulas

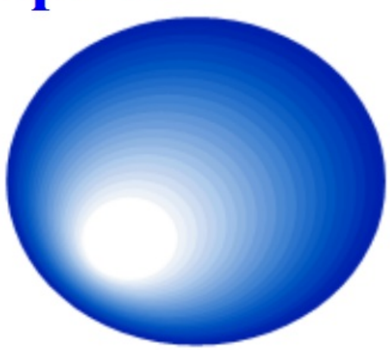
circle



$A = \pi r^2$

$C = 2\pi r$

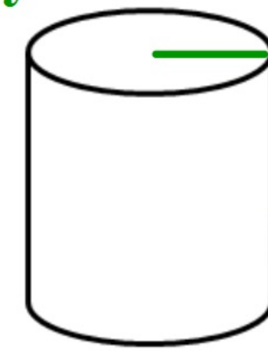
sphere



$V = \frac{4}{3}\pi r^3$

$SA = 4\pi r^2$

cylinder

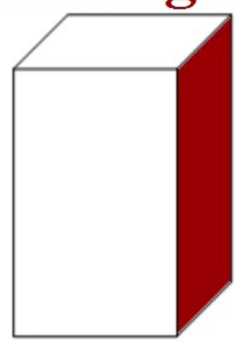


(can)

$V = \pi r^2 h$

$SA = 2\pi r h + \underbrace{2\pi r^2}_{\text{bottom + top of can}}$

rectangular prism *(with a square base)*

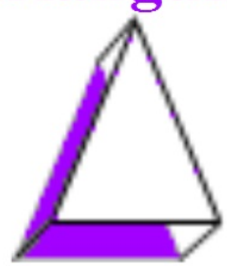


$V = l \cdot l \cdot h$

General $V = lwh$

$SA = 2l^2 + 4 \cdot h \cdot l$

triangular prism



$V = \frac{1}{2} b \cdot h \cdot d$ (depth)

1) A plot of land is bounded by a river on one side and the other three sides by a fence.

What ***dimensions*** should be used to ***minimize*** the amount of **fencing** required to enclose an **area of 500 yd^2**?

Given

$A = 500\,yd^2$

$A = \ell \cdot w$

$\ell \cdot w = 500$

$w = \frac{500}{\ell}$

Optimize

$P = 2w + \ell$

$P = 2\left(\frac{500}{\ell}\right) + \ell$

$P = 1000\,\ell^{-1} + \ell$

$P' = -1000\,\ell^{-2} + 1$

$0 = \frac{-1000}{\ell^2} + 1$

$\frac{1000}{\ell^2} = 1$

$1000 = \ell^2$

$\ell = \sqrt{1000}$

$w = \frac{500}{\sqrt{1000}}$

$w = \frac{500}{\sqrt{1000}} \approx 15.811$ yds.

$\ell = \sqrt{1000} \approx 31.623$ yds

The dimensions to minimize the amount of fencing should be $\ell = \sqrt{1000}$ yds. and $w = \frac{500}{\sqrt{1000}}$ yds.

2) Find the dimensions of a **rectangular prism** *(with a square base) without* a lid that will hold **1000 cm^3** and be made of the **least amount** of surface area.

Given

$V = 1000\text{ cm}^3$

$V = \ell^2 \cdot h$

$1000 = \ell^2 \cdot h$

$\frac{1000}{\ell^2} = h$

Optimizing

$SA = \ell^2 + 4h\ell$

$SA = \ell^2 + 4\left(\frac{1000}{\ell^2}\right) \cdot \ell$

$SA = \ell^2 + 4000\ell^{-1}$

$\frac{dSA}{d\ell} = 2\ell - 4000\ell^{-2}$

$0 = 2\ell - \frac{4000}{\ell^2}$

$\frac{4000}{\ell^2} = 2\ell$

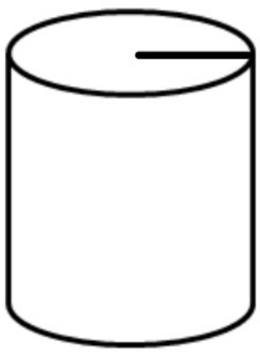
$2000 = \ell^3$

$\ell = \sqrt[3]{2000} \approx 12.599\text{cm}$

$h = \frac{1000}{\left(\sqrt[3]{2000}\right)^2} \approx 6.299\text{ cm.}$

The dimensions of the rect. prism w/o a lid are $\ell = \sqrt[3]{2000}$ cm.
$h = \frac{1000}{\left(\sqrt[3]{2000}\right)^2}$ cm.

3) Find the **dimensions** to optimize the **surface area** of a can **without a lid** that has a volume of **4000 cm^3**.



Given

$V = \pi r^2 h$

$4000 = \pi r^2 h$

$\frac{4000}{\pi r^2} = h$

Optimize

$SA = 2\pi r h + \pi r^2$

$SA = 2\pi r\left(\frac{4000}{\pi r^2}\right) + \pi r^2$

$SA = 8000 r^{-1} + \pi r^2$

$SA' = -8000 r^{-2} + 2\pi r$

$0 = -8000 r^{-2} + 2\pi r$

$\frac{8000}{r^2} = 2\pi r$

$\frac{8000}{2\pi} = r^3$

$10.839 \text{ cm.} \approx \sqrt[3]{\frac{4000}{\pi}} = r$

$h = \frac{4000}{\pi\left(\sqrt[3]{\frac{4000}{\pi}}\right)^2}$

$h \approx 10.839$ cm.

The dimensions to optimize the amount of surface area should be $r = \sqrt[3]{\frac{4000}{\pi}}$ cm. and $h = \frac{4000}{\pi\left(\sqrt[3]{\frac{4000}{\pi}}\right)^2}$ cm.