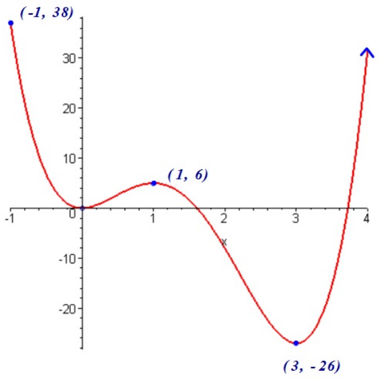
SECTION 5.1 EXTREME VALUES FOR A FUNCTION

First: **look at the top of page 193**

Given the graph of *h(x)*, identify the extreme values of the function



Note:
Points of interest at
$x=-1,0,1,3$

Minimum

relative (local):

h has a rel. min. of 0 at $x=0$ b/c h' changes from $-$ to $+$

absolute:

h has an abs. max of -26 at $x=3$ b/c h' changes from $-$ to $+$
and $h(0)=0$, $h(3)=-28$

Maximum

relative (local):

h has a rel. max of 38 at $x=-1$ b/c h' is negative after the endpt.
or b/c h decreases to the right of the endpt.

h has a rel. max of 6 at $x=1$ b/c h' changes from $+$ to $-$

absolute:

h does not have an abs. max b/c $\lim_{x\to\infty} f(x)=\infty$

Find the extreme values for the given functions. *(Be sure to complete the 5 step process)*

1) $f(x)=\frac{1}{3}x^3+x^2-15x+1$

a) Domain: $(-\infty,\infty)$

b) $\frac{dy}{dx}$: $f'(x)=x^2+2x-15$

c) $f'=0$

$x^2+2x-15=0$

$(x+5)(x-3)=0$

$x=-5,3$

$f'=$ undef.

never

points of interest at
$x=-5,3$

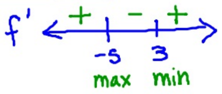
$f(-5)=\frac{1}{3}(-5)^3+(-5)^2-15(-5)+1$

$f(-5)=59.3333$

$f(3)=\frac{1}{3}(3)^3+(3)^2-15(3)+1$

$f(3)=-26$

d) # line



e) Answer

f has a rel. max of $\underset{}{\overset{f(5)}{\underline{59.\overline{3}}}}$ at $x=-5$ b/c f' changes from $+$ to $-$

f has a rel. min of $\overset{f(3)}{\underline{-26}}$ at $x=3$ b/c f' changes from $-$ to $+$

2) $f(x)=\frac{1}{3}x^3-4x^2+12x+5,\ 1\le x<9$

a) Domain $[1,9)$

b) $\frac{dy}{dx}$: $f'(x)=x^2-8x+12$

c) $f'=0$

$x^2-8x+12=0$

$(x-2)(x-6)=0$

$x=2,6$

$f'=$ undef.

never

points of interest at
$x=1,2,6,9$

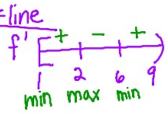
$f(1)=13.\overline{3}$

$f(2)=15.\overline{6}$

$f(6)=5$

$f(9)=32$

d) # line



e) answer

f has a rel. max of $\overset{f(2)}{\underline{15.\overline{6}}}$ at $x=2$ b/c f' changes from $+$ to $-$

f has a rel. min of $\overset{f(1)}{\underline{13.\overline{3}}}$ @ $x=1$ b/c f' is $+$ after the endpt.

f has an abs. min of $\overset{f(6)}{\underline{5}}$ @ $x=6$ b/c f' changes from $-$ to $+$
and $f(1)=13.3$
$f(6)=5$

3) $f(x) = \frac{1}{x^2 - 1}$

a) Domain: $(-\infty, -1)(-1, 1)(1, \infty)$

b) $\frac{dy}{dx}$: $f' = \frac{(x^2-1)(0) - (1)(2x)}{(x^2-1)^2}$

$f' = \frac{-2x}{(x^2-1)^2}$

c) $\underline{f' = 0}$

$-2x = 0$

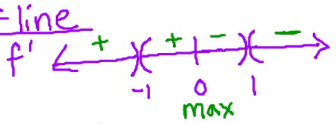
$x = 0$

$\underline{f' = \text{undef.}}$

$x^2 - 1 = 0$

$x = \pm 1$

d) $\underline{\#\text{line}}$



point of interest @ $x = 0$

$\boxed{f(0) = -1}$

e) $\underline{\text{answer}}$

f has a relative max of $\underset{}{\overset{f(0)}{\underline{-1}}}$ at $x=0$ b/c f' changes from + to −

f does not have a min.

4) $g(x) = \frac{1}{\sqrt{4 - x^2}} = (4 - x^2)^{-1/2}$

a) Domain: $(-2, 2)$

b) $\frac{dy}{dx}$: $g' = -\frac{1}{2}(4 - x^2)^{-3/2}(-2x)$

$g' = \frac{x}{(4 - x^2)^{3/2}}$

c) $\underline{g' = 0}$

$x = 0$

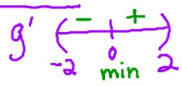
$\underline{g' = \text{undef.}}$

$4 - x^2 = 0$

$x^2 = 4$

$x = \pm 2$

d) $\underline{\#\text{line}}$



point of interest at $x = 0$

$\boxed{f(0) = \frac{1}{2}}$

e) $\underline{\text{answer}}$

g has an abs min of $\overset{f(0)}{\underline{\frac{1}{2}}}$ at $x=0$ b/c g' changes from − to +

g does not have a max.

5) $y = \cos\left(x - \frac{\pi}{3}\right), \quad 0 < x \le \frac{5\pi}{3}$

a) Domain: $\left(0, \frac{5\pi}{3}\right]$

b) $\frac{dy}{dx}$: $y' = -\sin\left(x - \frac{\pi}{3}\right)$

c) $\underline{y' = 0}$

$-\sin\left(x - \frac{\pi}{3}\right) = 0$

$\sin\left(x - \frac{\pi}{3}\right) = 0$

$\underline{y' = \text{undef}}$

never

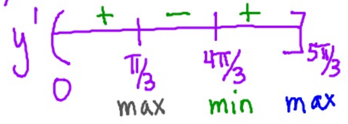
$\sin x = 0$

$x = 0, \pi, 2\pi$

$\left(x - \frac{\pi}{3}\right) = 0, \pi, 2\pi$

$x = \pi/3, 4\pi/3$

d) $\underline{\#\text{line}}$



points of interest
$x = 0, \pi/3, 4\pi/3, 5\pi/3$

$$\boxed{\begin{array}{l} f(0) = \frac{1}{2} \\ f(\pi/3) = 1 \ ^{\text{max}} \\ f(4\pi/3) = -1 \ ^{\text{min}} \\ f(5\pi/3) = -\frac{1}{2} \ ^{\text{max}} \end{array}}$$

e) $\underline{\text{Answer}}$

y has an abs. max of $\underline{1}$ at $x = \pi/3$
b/c y' changes from + to − and $f(\pi/3) = 1$ and $f\left(\frac{5\pi}{3}\right) = -\frac{1}{2}$

y has a rel. max of $\underline{-\frac{1}{2}}$ at $x = 5\pi/3$
b/c y' is + before the endpt.
(OR b/c y increases before the endpt.)

y has an abs. min of $\underline{-1}$ at $x = 4\pi/3$
b/c y' changes from − to +