Connecting 1st and 2nd derivatives with the original function

Section 5.3

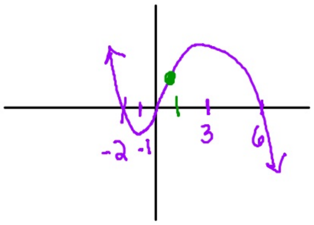
1st Derivative: Relative and absolute
- maximums
- minimums

Increasing intervals
Decreasing intervals

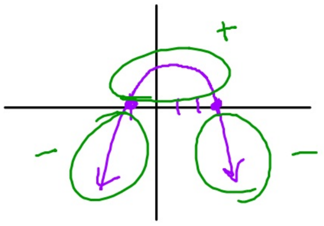
2nd Derivative: • Concavity
concave up ↺
concave down ↻

• Point of inflection
where the original graph changes concavity

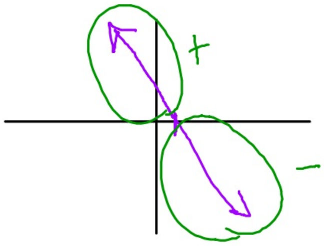
$f(x) =$



$f'(x) =$



$f''(x) =$



Info. about f

min @ $x=-1$

max @ $x=3$

f is inc. on $(-1,3)$

f is dec. on $(-\infty,-1) \cup (3,\infty)$

Info. about f

f is cc↑ $(-\infty,1)$

f is cc↓ $(1,\infty)$

pt. of inf at $x=1$

1) $y = 4x^3 - 12x^2$

a) Domain: $(-\infty, \infty)$

b) $\frac{dy}{dx}$ $y' = 12x^2 - 24x$

c) C.P. $y' = 0$ $y' =$ undef

never

$12x^2 - 24x = 0$

$12x(x-2) = 0$

$x = 0, 2$

d) # line

max min

$y'\ (\ + \mid - \mid +\ )$

0 2

points of interest

$y(0) = 0$

$y(2) = -16$

$y(1) = -8$

b) $\frac{d^2y}{dx^2}$ $y'' = 24x - 24$

c) C.P. $y'' = 0$ $y'' =$ undef

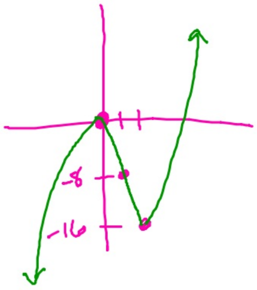
never

$24x - 24 = 0$

$x = 1$

d) # line $y''\ (\ - \mid +\ )$

CC↓ 1 CC↑



e) Answer

y is increasing on $(-\infty, 0)(2, \infty)$ b/c $y' > 0$

y is decreasing on $(0, 2)$ b/c $y' < 0$

y has a rel. max of 0 at $x = 0$ b/c y' changes from + to −

y has a rel. min of −16 at $x = 2$ b/c y' changes from − to +

y is CC↑ on $(1, \infty)$ b/c $y'' > 0$

y is CC↓ on $(-\infty, 1)$ b/c $y'' < 0$

y has a p.o.i. at $(1, -8)$ b/c y'' changes signs

2) $y = xe^x$

a) Domain $(-\infty, \infty)$

b) $\frac{dy}{dx}$ $y' = x \cdot e^x + (1) \cdot e^x$

$y' = e^x(x+1)$

c) C.P. $y' = 0$ $y' =$ undef.

never

$0 = e^x(x+1)$

$e^x = 0$ ~~never~~ $x + 1 = 0$

$x = -1$

d) # line

min

$y'\ (\ - \mid +\ )$

−1

Points of interest

$y(-1) = -e^{-1} = -\frac{1}{e} \approx -0.368$

$y(-2) = -2e^{-2} = \frac{-2}{e^2} \approx -0.271$

$y' = \underbrace{x e^x}{} + \underbrace{e^x}{}$

b) $y'' = xe^x + e^x + e^x$

$y'' = xe^x + 2e^x$

$y'' = e^x(x+2)$

c) C.P. $y'' = 0$ $y'' =$ undef.

never

$\underbrace{e^x}_{\text{never}}(x+2) = 0$

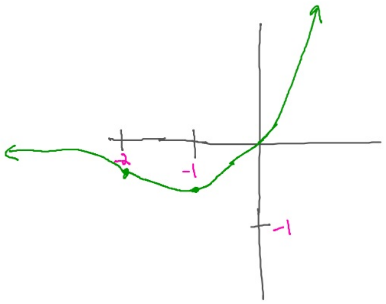
$x = -2$

d) # line

p.o.i.

$y''\ (\ - \mid +\ )$

−2



e) Answer

y is incr. on $(-1, \infty)$ b/c $y' > 0$

y is decr. on $(-\infty, -1)$ b/c $y' < 0$

y has an abs. min of $-\frac{1}{e}$ at $x = -1$ b/c y' changes from − to +

and

y does not have a max

y is CC↑ on $(-2, \infty)$ b/c $y'' > 0$

y is CC↓ on $(-\infty, -2)$ b/c $y'' < 0$

y has a p.o.i. at $(-2, -2e^{-2})$ b/c y'' changes signs