Implicit Differentiation

Section 4.2

$\frac{d}{dx}(y) = \frac{dy}{dx}$

$\frac{d}{dx}(y^5) = 5y^4 \cdot \frac{dy}{dx}$

$\frac{d}{dx}(x) = \frac{dx}{dx} = 1$

$\frac{d}{dx}(3x) = 3\frac{dx}{dx} = 3$

$\frac{d}{dx}(4x^7) = 28x^6 \cdot \frac{dx}{dx} = 28x^6$

Find $\frac{dy}{dx}$

1) $y = x^2 - 3x + 2$

$\frac{dy}{dx} = 2x - 3$

2) $5 = 2x + 3y - x^2$

$0 = 2 + 3\frac{dy}{dx} - 2x$

$-3\frac{dy}{dx} = 2 - 2x$

$\frac{dy}{dx} = -\frac{2}{3} + \frac{2}{3}x$

3) $4x = 7x^2 - \frac{3x^5}{5} + 9y^3$

$4 = 14x - 3x^4 + 27y^2 \cdot \frac{dy}{dx}$

$4 - 14x + 3x^4 = 27y^2 \cdot \frac{dy}{dx}$

$\frac{4 - 14x + 3x^4}{27y^2} = \frac{dy}{dx}$

Review $y = \cos(2x)$

$\frac{dy}{dx} = -\sin(2x) \cdot 2$

4) $x + \cos y = 5$

$1 - \sin y \cdot \frac{dy}{dx} = 0$

$-\sin y \cdot \frac{dy}{dx} = -1$

$\frac{dy}{dx} = \frac{1}{\sin y}$

$\frac{dy}{dx} = \csc y$

5) $\tan x - x^2 \cdot y = 4$

$\sec^2 x - \left(x^2 \cdot \frac{dy}{dx} + y \cdot 2x\right) = 0$

$\sec^2 x - x^2 \cdot \frac{dy}{dx} - 2xy = 0$

$-x^2 \cdot \frac{dy}{dx} = 2xy - \sec^2 x$

$\frac{dy}{dx} = \frac{2xy - \sec^2 x}{-x^2}$

6) $5xy - 3x^2 y = \pi^4$

$5x \cdot \frac{dy}{dx} + y \cdot 5 - \left(3x^2 \cdot \frac{dy}{dx} + y \cdot 6x\right) = 0$

$\underline{5x\frac{dy}{dx}} + 5y - \underline{3x^2\frac{dy}{dx}} - 6xy = 0$

$\frac{dy}{dx}(5x - 3x^2) = 6xy - 5y$

$\frac{dy}{dx} = \frac{6xy - 5y}{5x - 3x^2}$

7) $3x - 2xy + \sin y = 5$

$3 - \left(2x\frac{dy}{dx} + y \cdot 2x\right) + \cos y \cdot \frac{dy}{dx} = 0$

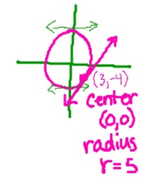
$3 - \underline{2x\frac{dy}{dx}} + 2xy + \underline{\cos y \cdot \frac{dy}{dx}} = 0$

$\frac{dy}{dx}(-2x + \cos y) = -2xy - 3$

$\frac{dy}{dx} = \frac{-2xy - 3}{-2x + \cos y}$

8) $x^2+y^2=25$ a) Find the tangent line at (3,-4)

b) Find the tangent line parallel to the x-axis



a) pt.

(3,-4)

slope

$2x+2y\cdot\frac{dy}{dx}=0$

$2(3)+2(-4)\frac{dy}{dx}=0$

$6-8\frac{dy}{dx}=0$

$\frac{dy}{dx}=\frac{6}{8}$

$\frac{dy}{dx}=\frac{3}{4}$

$\boxed{y+4=\frac{3}{4}(x-3)}$

b) pt.

(0, ?)

$(0)^2+y^2=25$

$y^2=25$

$y=\pm5$

(0,5) (0,-5)

slope

$\frac{dy}{dx}=m=0$

$2x+2y(0)=0$

$2x=0$

$x=0$

$y-5=0(x-0)$

$y=5$

$y+5=0(x-0)$

$y=-5$

9) Find the tangent to the curve $x^2-xy+y^2=7$ at (-1,2)

pt.

(-1,2)

Slope

$2x-\left(x\cdot\frac{dy}{dx}+y\cdot1\right)+2y\cdot\frac{dy}{dx}=0$

$2x-x\cdot\frac{dy}{dx}-y+2y\cdot\frac{dy}{dx}=0$

$2(-1)-(-1)\frac{dy}{dx}-(2)+2(2)\cdot\frac{dy}{dx}=0$

$-2+\frac{dy}{dx}-2+4\frac{dy}{dx}=0$

$5\frac{dy}{dx}=4$

$\frac{dy}{dx}=\frac{4}{5}$

Review:

Find the inverse of: $f(x)=2x-4$

$x=2y-4$

$\frac{x+4}{2}=y$

$f^{-1}(x)=\frac{1}{2}x+2$

Simplify: $\frac{d}{dx}(f[g(x)])=$ $f'(g(x))\cdot g'(x)$

What can you say about the composition of a function and it's inverse?

$h(h^{-1}(x))=x$ $h^{-1}(h(x))=x$

10) Find the slope of the inverse of any function

Given $h(h^{-1}(x))=x$

$\frac{d}{dx}\left[h(h^{-1}(x))\right]=\frac{d}{dx}[x]$

$h'[h^{-1}(x)]\cdot\frac{d}{dx}[h^{-1}(x)]=1$

$\frac{d}{dx}[h^{-1}(x)]=\frac{1}{h'[h^{-1}(x)]}$