DERIVATIVES OF EXPONENTIAL & LOGARITHMIC FUNCTIONS

SECTION 4.4

Rule for exponents

$\frac{d}{dx}(a^u) = a^u \cdot \ln a \cdot \frac{du}{dx}$

Exponential:

1) $y = 2^x$

$y' = 2^x \cdot \ln 2 \cdot (1)$

$y' = 2^x \ln 2$

2) $y = 5^{4x+2}$

$y' = 5^{4x+2}(\ln 5)(4)$

3) $y = 7^{\pi - 2x^3}$

$y' = 7^{\pi-2x^3} \cdot \ln 7 \cdot (-6x^2)$

4) $y = e^x$

$y' = e^x \cdot \ln e \cdot 1$

$y' = e^x$

5) $y = e^{\cos(2x+1)}$

$y' = e^{\cos(2x+1)} \cdot 1 \cdot (-\sin(2x+1)(2))$

$y' = -2\sin(2x+1)e^{\cos(2x+1)}$

6) $y = e^{\pi - 4x}$

$y' = e^{\pi-4x} \cdot 1 \cdot (-4)$

$y' = -4e^{\pi-4x}$

7) $y = 5^{e+1}$

$y' = 0$

Logs:

Rule for logs

$\frac{d}{dx}(\log_a u) = \frac{1}{u} \cdot \frac{1}{\ln a} \cdot \frac{du}{dx}$

8) $y = \log_5(2x^2 + 4x)$

$y' = \frac{1}{2x^2+4x} \cdot \frac{1}{\ln 5} \cdot (4x+4)$

9) $y = \log x^2$

$y' = \frac{1}{x^2} \cdot \frac{1}{\ln 10} \cdot (2x) = \frac{2}{x \ln 10}$

10) $y = \ln t$

$y' = \frac{1}{t} \cdot \frac{1}{\ln e} \cdot 1$

$y' = \frac{1}{t}$

11) $y = \ln(3a^2 - 5)$

$y' = \frac{1}{3a^2-5} \cdot \frac{1}{\ln e} \cdot (6a)$

$y' = \frac{6a}{3a^2-5}$

12) $y = x^{1+\sqrt{2}}$

$y' = (1+\sqrt{2})x^{\sqrt{2}}$

Logarithmic Differentiation

13) $y = x^x$

$\ln(y) = \ln(x^x)$

$\ln y = x \cdot \ln x$

$\frac{1}{y} \cdot \frac{1}{\ln e} y' = \left[x \cdot \frac{1}{x} \cdot \frac{1}{\ln e} \cdot (1)\right] + \left[\ln x \cdot 1\right]$

$\frac{1}{y} y' = 1 + \ln x$

$y' = y(1 + \ln x)$

$y' = x^x(1 + \ln x)$

14) $y = x^{\ln x}$

$\ln y = \ln x^{\ln x}$

$\ln y = \ln x \cdot \ln x$

$\frac{1}{y} \cdot y' = \ln x \cdot \frac{1}{x} + \ln x \cdot \frac{1}{x}$ $= 2 \ln x \cdot \frac{1}{x}$

$\frac{1}{y} \cdot y' = \frac{2}{x} \ln x$

$y' = y\left(\frac{2}{x} \ln x\right)$

$y' = x^{\ln x}\left(\frac{2}{x} \ln x\right)$