Important Things to Know
Chapter 3 Test

Limits

- Definition of a limit

Continuity

- Three criteria for continuity at a point
  - $f(c)$ exists
  - $\lim_{x \to c^+} f(x) = \lim_{x \to c^-} f(x)$
  - $f(c) = \lim_{x \to c} f(x)$

Differentiability

- What does differentiability mean? possible to find the deriv. at a point
- Make a piecewise function continuous and differentiable *(similar to problem 39b on p 115)*

$f(x) = \begin{cases} 3-x & x<1 \\ ax^2+bx & x\geq 1 \end{cases}$

$f'(x) = \begin{cases} -1 & x<1 \\ 2ax+b & x\geq 1 \end{cases}$

$3-x = ax^2+bx$
$3-(1) = a(1)^2+b(1)$
$2 = a+b$
$2-a = b$

$-1 = 2ax+b$
$-1 = 2a(1)+b$
$-1 = 2a+b$
$-1 = 2a+(2-a)$
$-1 = a+2$
$-3 = a$

$2-(-3) = b$
$5 = b$

Rules for Derivatives

- Power rule $\frac{d}{dx} u^n \to nu^{n-1}$
- Product rule $\frac{d}{dx}(uv) = u \cdot v' + v u'$
- Quotient rule $\frac{d}{dx}\left(\frac{u}{v}\right) = \frac{v \cdot u' - uv'}{v^2}$
- Derivatives of Trigonometric Functions

$f(x)$	$f'(x)$	$f(x)$	$f'(x)$
$\sin(x)$	$\cos x$	$\csc(x)$	$-\cot x \csc x$
$\cos(x)$	$-\sin x$	$\sec(x)$	$\tan x \sec x$
$\tan(x)$	$\sec^2 x$	$\cot(x)$	$-\csc^2 x$

Position / Velocity / Acceleration

$s(t)$ = position
$s'(t) = v(t)$ = velocity
$s''(t) = v'(t) = a(t)$ = acceleration

Velocity — direction, speed

Forward / Upward motion: $v(t) > 0$
Backward / Downward motion: $v(t) < 0$

Speed $|v|$

Displacement = $s(b) - s(a)$

Average Velocity = $\frac{s(b)-s(a)}{b-a}$ $\frac{ft}{sc}$

Instantaneous Velocity = $v(t) = s'(t)$

Acceleration
Speed-up: v and a are the same sign
Slow-down: v and a are opposite signs

CH 3 Class Review

An object is propelled upward with an initial velocity of 16 feet per second so that its height is given by $h(t) = -.8t^2 + 16t$.

a. Find the objects velocity and acceleration at any time t.
b. When did the object reach its maximum height?
c. What was the maximum height of the object?
d. When did the object reach half its maximum height?
e. When did the object hit the ground?

$h(t) = 0$
$-0.8t^2 + 16t = 0$
$t(-0.8t + 16) = 0$
$-0.8t + 16 = 0$
$16 = 0.8t$
$20 = t$
20 Seconds

a) $h(t) = -0.8t^2 + 16t$
$h'(t) = V(t) = -1.6t + 16$
$h''(t) = v'(t) = a(t) = -1.6$

b) $0 = V(t)$
$0 = -1.6t + 16$
$1.6t = 16$
$t = 10$ seconds

c) $h(10) = -0.8(10)^2 + 16(10)$
$h(10) = -80 + 160$
$h(10) = 80$ ft.

d) $40 = -0.8t^2 + 16t$
$0.8t^2 - 16t + 40 = 0$
$t = \frac{16 \pm \sqrt{(-16)^2 - 4(0.8)(40)}}{2(0.8)}$
$t = 10 \pm 5\sqrt{2}$
$t \approx 2.929$ seconds

The number of gallons in a water tank in t minutes is given by $V(t) = 300(25-t)^2$. The tank is being drained. How fast is the tank draining at 3 minutes? 7 minutes? When will the tank be empty?

$$V(t) = 300(25-t)^2 \quad \text{gallons}$$

$$V(t) = 300(25-t)(25-t)$$

$$V(t) = 300(625 - 50t + t^2)$$

$$V(t) = 187500 - 15000t + 300t^2$$

$$V'(t) = v(t) = -15{,}000 + 600t$$

$$V'(3) = v(3) = -15{,}000 + 600(3)$$

$$= -13{,}200 \text{ gallons/min}$$

speed is 13,200 gall/min