Section 3.3 Rules for Derivatives

Rules:
- Power Rule
- Product Rule
- Quotient Rule

Power Rule

$f(x)=x^n$

$f'(x)=nx^{n-1}$

Find the derivative.

1. $y=\frac{4}{x^4}-\frac{3}{x^3}+2x^2-\frac{1}{x}+x+2$

$y=4x^{-4}-3x^{-3}+2x^2-x^{-1}+x+2$

$y'=-16x^{-5}+9x^{-4}+4x+x^{-2}+1$

2. $y=\frac{x^2}{3}$

$y=\frac{1}{3}x^2$

$y'=\frac{2}{3}x$

3. $y=\frac{5}{\sqrt{x}}$

$y=5x^{-\frac{1}{2}}$

$y'=-\frac{5}{2}x^{-\frac{3}{2}}$

Product Rule

$\frac{d}{dx}(u\cdot v)=u\cdot v'+v\,u'$

"first d'last + last d'first"

4. $f(x)=(5x^4-3x)(6x^{-2})$

$f'(x)=(5x^4-3x)(-12x^{-3})+(6x^{-2})(20x^3-3)$

$f'(x)=-60x+36x^{-2}+120x-18x^{-2}$

$=60x+18x^{-2}$

$=60x+\frac{18}{x^2}$

$f(x)=u\cdot v \qquad f'(x)=uv'+vu'$

5. $f(x)=(x^2-3x)(x^3-4)$

$f'(x)=(x^2-3x)(3x^2)+(x^3-4)(2x-3)$

$f'(x)=3x^4-9x^3+2x^4-8x-3x^3+12$

$f'(x)=5x^4-12x^3-8x+12$

Quotient Rule

$\frac{d}{dx}\left(\frac{u}{v}\right)=\frac{v\cdot u'-u\cdot v'}{v^2}$

"low d'high - high d'low all over low squared"

6. $y=\frac{1}{x+2}$

$y'=\frac{(x+2)(0)-(1)(1)}{(x+2)^2}$

$y'=\frac{-1}{(x+2)^2}$

7. $f(x)=\frac{x+1}{x^2-4}$

$-2x(x+1)$

$f'(x)=\frac{(x^2-4)(1)-(x+1)(2x)}{(x^2-4)^2}$

$f'(x)=\frac{x^2-4-2x^2-2x}{(x^2-4)^2}$

$f'(x)=\frac{-x^2-2x-4}{(x^2-4)^2}$

Given:

$f(2)=3$

$f'(2)=-1$

$g(2)=4$

$g'(2)=2$

8. If $h(x)=f(x)\cdot g(x)$, find $\frac{dh}{dx}(2)$.

$h'(2)=f(2)\cdot g'(2)+g(2)\cdot f'(2)$

$h'(2)=3\cdot 2+4\cdot(-1)$

$h'(2)=2$

9. If $h(x)=\frac{f(x)}{g(x)}$, find $\frac{dh}{dx}(2)$.

$h'(2)=\frac{g(2)\cdot f'(2)-f(2)\cdot g'(2)}{[g(2)]^2}$

$\boxed{g(2)^2 \neq [g(2)]^2}$

$h'(2)=\frac{4\cdot(-1)-(3)\cdot(2)}{(4)^2}$

$h'(2)=\frac{-5}{8}$

10. $\frac{d}{dx}(3f-5g)$ at $x=2$

$=3f'(2)-5g'(2)$

$=3(-1)-5(2)$

$=-3-10$

$=-13$

$\boxed{\begin{array}{l}\frac{d}{dx}3f\\ 3f'+f\cdot(0)\end{array}}$

11. Given $f(x)=x^2-x-6$, find the tangent line parallel to the x-axis.

pt.

$\left(\frac{1}{2}, f\left(\frac{1}{2}\right)\right)$

$f\left(\frac{1}{2}\right)=\left(\frac{1}{2}\right)^2-\left(\frac{1}{2}\right)-6$

$=\frac{1}{4}-\frac{1}{2}-6$

$f\left(\frac{1}{2}\right)=-\frac{25}{4}$

$\left(\frac{1}{2}, -\frac{25}{4}\right)$

slope

$m=0$

$f'(x)=2x-1$

$2x-1=0$

$x=\frac{1}{2}$

$y=-\frac{25}{4}$ OR $y+\frac{25}{4}=0\left(x-\frac{1}{2}\right)$