Sec 3.4 Velocity

$s(t) =$ position function

$s'(t) = v(t) =$ velocity function

$s''(t) = v'(t) = a(t) =$ acceleration function

Velocity → speed
→ direction

Forward / Upward motion: $v(t) > 0$

Backward / Downward motion: $v(t) < 0$

Speed = $|v| = \left|\frac{ds}{dt}\right|$ (derivative of $s(t)$ w/respect to time)

Displacement = Δposition = $s(\text{end time}) - s(\text{begin time})$

Average Velocity = $\frac{\Delta \text{position}}{\Delta \text{time}} = \frac{s(\text{end time}) - s(\text{begin time})}{\text{end time} - \text{begin time}}$

Instantaneous Velocity = $s'(t) = v(t)$

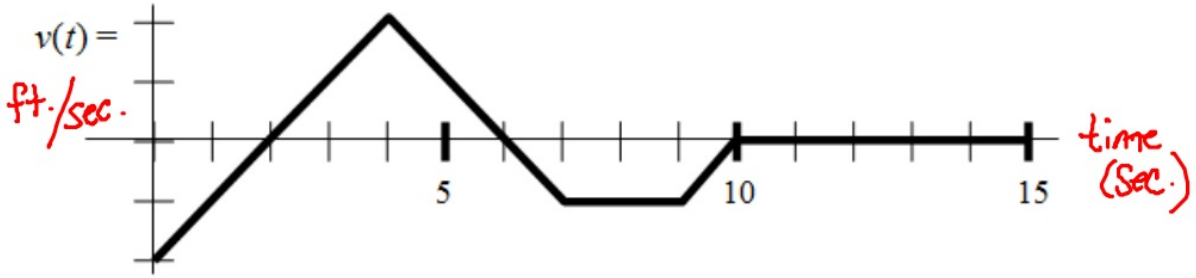
Acceleration

Speed-up: $v(t)$ and $a(t)$ are the same sign

Slow-down: $v(t)$ and $a(t)$ are opposite signs

1) An ant in motion

The graph shows the *velocity* V *ft/sec.* of an ant crawling on a number line



a) When does the ant change direction?

The ant changes direction at $t = 2$ and 6 seconds b/c $v(t)$ changes signs

b) When does the ant move forward?

The ant moves forward between $(2,6)$ seconds b/c $v > 0$

When does the ant move backward?

The ant moves backward between $[0,2) \cup (6,10)$ seconds b/c $v < 0$

c) What is the speed of the ant at 3 seconds? 1 ft/sec.

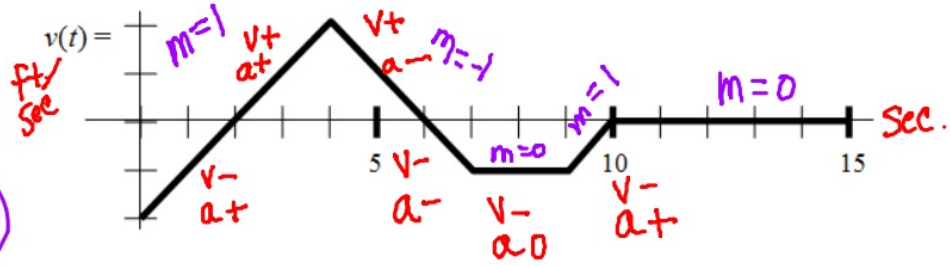
d) What is the speed of the ant at 4 seconds? 2 ft/sec.

e) What is the speed of the ant at 8 seconds? 1 ft/sec.

f) When is the ant moving the fastest?

$t=0$ and 4 seconds

(v is furthest from the t-axis)



g) When does the ant speed up?

The ant speeds up $(2,4)$ and $(6,7)$ b/c $V(t)$ and $a(t)$ are the same signs

When does the ant slow down?

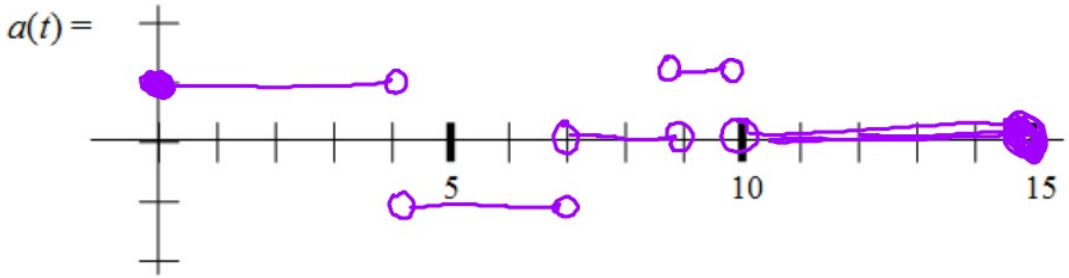
The ant slows down $[0,2) \cup (4,6) \cup (9,10)$ b/c v and a are opposite signs

When does the ant move at a constant speed?

The ant moves at a constant speed $(7,9)\,(10,15]$

b/c $a(t)=0$

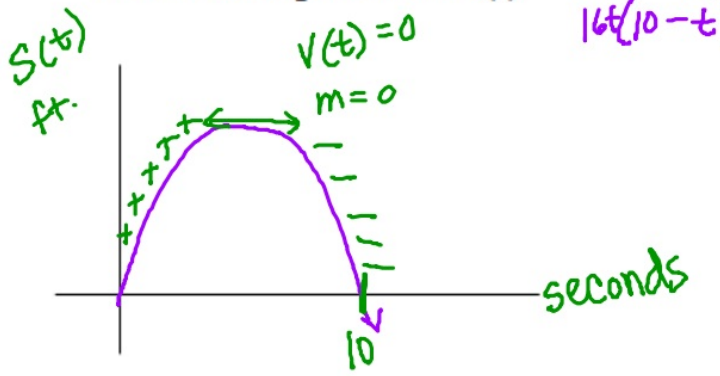
h) Graph the acceleration of the ant.



2. Example problem on p.130-131 #4

Dynamite propels a rock straight up with a velocity of 160 ft/sec.

The rock's position $s(t) = 160t - 16t^2$ *(ft.)*



$16t(10-t)$

$s'(t) = v(t)$

$s''(t) = v'(t) = a(t)$

a) When does the rock hit the ground?

$s(t) = 0$

$160t - 16t^2 = 0$

$16t(10-t) = 0$

$16t = 0 \quad 10 - t = 0$

$t = 0 \quad t = 10$

The rock hits the ground at $t = 10$ seconds

b) How high does the rock go?

1st: $s'(t) = 0$

$160 - 32t = 0$

$160 = 32t$

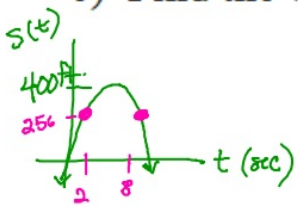
$5 = t$

$s(5) = 160(5) - 16(5)^2$

$s(5) = 400$ ft.

The rock reaches a max. height of 400 ft.

c) Find the **velocity** of the rock at **256 ft.**



1st: Find when the rock is 256ft. in the air

$$256 = 160t - 16t^2$$

$$16t^2 - 160t + 256 = 0$$

$$16(t^2 - 10t + 16) = 0$$

$$16(t-2)(t-8) = 0$$

$$t = 2, 8$$

$s'(t) = v(t) = 160 - 32t$

2nd: Plug the time into the v(t) function

$v(2) = 160 - 32(2)$ $\quad$ $v(8) = 160 - 32(8)$

$v(2) = 96$ ft/sec $\quad$ $v(8) = -96$ ft/sec

d) Find the rock's acceleration at any given time

$$s''(t) = v'(t) = a(t) = -32 \text{ ft/se}$$

3. **A ball is rolling along the *x*-axis according to the function**

$s(t) = \frac{1}{3}t^3 - 3t^2 + 8t - 10$ *(ft.)*

Start working on this problem . . . .
graph s(t) with your calculator

a) When does the ball change direction?

b) When is the ball moving right?

When is the ball moving left?