Section 2.4 Rates of Change & Tangent Lines

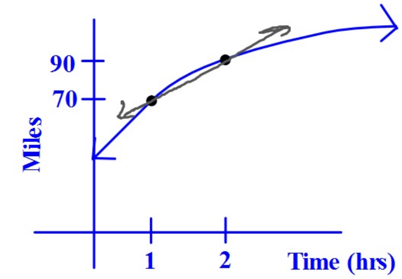
Average Rate of Change: $\dfrac{f(b)-f(a)}{b-a}$ Slope over an interval

secant line

Instantaneous rate of change: $\displaystyle\lim_{h\to 0}\frac{f(x+h)-f(x)}{(x+h)-x}$ Slope at a specific time (x-value)

tangent line

Writing equations of tangent/normal lines:

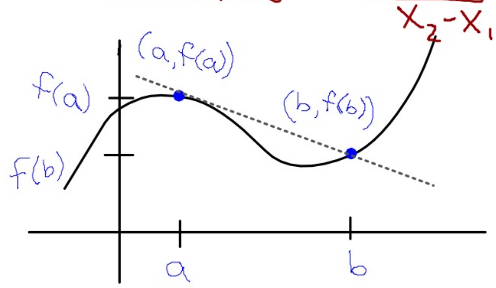


Ave. Rate of change

$\dfrac{90-70}{2-1} = 20$ miles/hr

These all mean rate of change:

- Slope
- $\dfrac{\text{rise}}{\text{run}}$
- m
- derivative
- velocity
- $\dfrac{\Delta y}{\Delta x}$
- $\dfrac{\Delta f}{\Delta x}$
- $\dfrac{y_2-y_1}{x_2-x_1}$



Average Rate of Change

$$\frac{f(b)-f(a)}{b-a}$$

Find the average rate of change over the interval.

1) $f(x)=\sqrt{4x+1}$ $[\overset{a}{0},\overset{b}{2}]$

$$m=\frac{f(2)-f(0)}{2-0}$$

$$=\frac{\sqrt{4(2)+1}-\sqrt{4(0)+1}}{2}$$

$$=\frac{3-1}{2}$$

$$=\boxed{1}$$

2) $f(x)=2+\cos x$ $[\overset{a}{0},\overset{b}{\pi}]$

$$m=\frac{f(\pi)-f(0)}{\pi-0}$$

$$=\frac{(2+\cos\pi)-(2+\cos 0)}{\pi-0}$$

$$=\frac{(2+-1)-(2+1)}{\pi}$$

$$=\boxed{-\frac{2}{\pi}}$$

3) $f(x)=x^2-x-6$ $[0,4]$

$$m=\frac{f(4)-f(0)}{4-0}$$

$$=\frac{(4^2-4-6)-(0^2-0-6)}{4}$$

$$=\frac{6--6}{4}$$

$$=\boxed{3}$$

Review

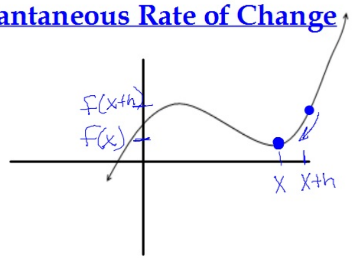
$f(x) = x^2 + 3x - 1$

$f(1+m) = (1+m)^2 + 3(1+m) - 1$

$= 1 + 2m + m^2 + 3 + 3m - 1$

$= m^2 + 5m + 3$

Instantaneous Rate of Change



$$\lim_{h\to 0} \frac{f(x+h) - f(x)}{(x+h) - x}$$

Find the Instantaneous Rate of Change

4. $f(x) = 2x^2 - 1$ at $x = 4$

$$= \lim_{h\to 0} \frac{f(4+h) - f(4)}{(4+h) - 4}$$

$$= \lim_{h\to 0} \frac{[2(4+h)^2 - 1] - [2(4)^2 - 1]}{h}$$

$$= \lim_{h\to 0} \frac{2(16 + 8h + h^2) - 1 - (2\cdot 16 - 1)}{h}$$

$$= \lim_{h\to 0} \frac{\cancel{32} + 16h + 2h^2 \cancel{-1} \cancel{-31}}{h}$$

$$= \lim_{h\to 0} \frac{\cancel{h}(16 + 2h)}{\cancel{h}}$$

$$= 16 + 2(0)$$

$$= \boxed{16}$$

5. $f(x) = x^2 - x - 2$ at $x = 3$

$$= \lim_{h\to 0} \frac{f(3+h) - f(3)}{(3+h) - 3}$$

$$= \lim_{h\to 0} \frac{[(3+h)^2 - (3+h) - 2] - [(3)^2 - (3) - 2]}{h}$$

$$= \lim_{h\to 0} \frac{[9 + 6h + h^2 - 3 - h - 2] - [4]}{h}$$

$$= \lim_{h\to 0} \frac{h^2 + 5h}{h}$$

$$= \lim_{h\to 0} \frac{\cancel{h}(h+5)}{\cancel{h}} = 0 + 5 = \boxed{5}$$

Instantaneous Rate of Change at any Point

6. $f(x) = x^2 - x - 2$ at any point x

$$= \lim_{h\to 0} \frac{f(x+h) - f(x)}{(x+h) - x}$$

$$= \lim_{h\to 0} \frac{[(x+h)^2 - (x+h) - 2] - (x^2 - x - 2)}{h}$$

$$= \lim_{h\to 0} \frac{\cancel{x^2} + 2xh + h^2 \cancel{- x} - h \cancel{- 2} \cancel{- x^2} \cancel{+ x} \cancel{+ 2}}{h}$$

$$= \lim_{h\to 0} \frac{\cancel{h}(h + 2x - 1)}{\cancel{h}}$$

$$= \boxed{2x - 1}$$

normal line is $\perp$ to tangent line

7. *Find the equation of the line tangent at x=3 for* $f(x) = x^2 - x - 2$

Point: $(3,4)$

$f(3) = 3^2 - 3 - 2$

slope: $m = 5$ from #5

$$\boxed{y - 4 = 5(x - 3)}$$

8. *Find the equation of the line normal at x=3 for* $f(x) = x^2 - x - 2$

Point: $(3,4)$

slope: $m = -\frac{1}{5}$

$$\boxed{y - 4 = -\frac{1}{5}(x - 3)}$$