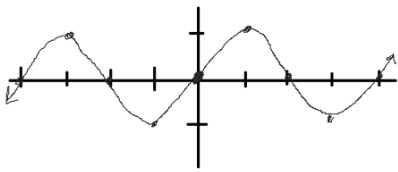
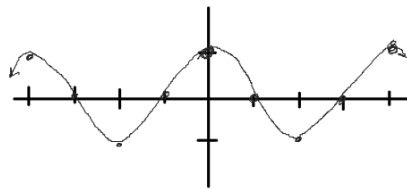


Trig Graphs

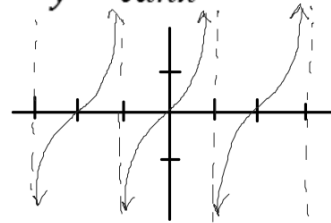
$$y = \sin x$$



$$y = \cos x$$



$$y = \tan x$$



Identify parts of the equation

$$y = 2 \sin(3x + \pi) + 4$$

amplitude: $\frac{2 \sin 3(x + \frac{\pi}{3}) + 4}{2}$

period: $\frac{2\pi}{3}$

phase shift (H.S.): Left $+\frac{\pi}{3}$

vertical shift: up 4

$$y = -3 \cos(5x - 4)$$

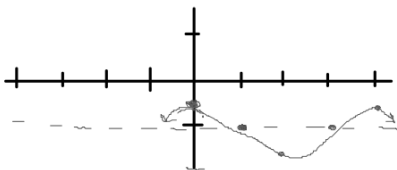
amplitude: $\frac{-3 \cos 5(x - \frac{4}{5})}{3}$ (reflection)

period: $\frac{2\pi}{5}$

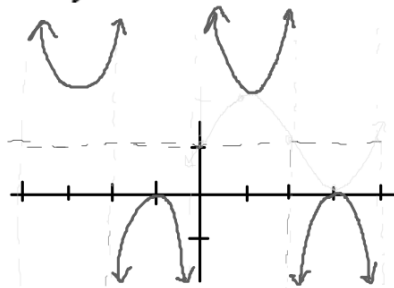
phase shift (H.S.): right $\frac{4}{5}$

vertical shift: none or 0

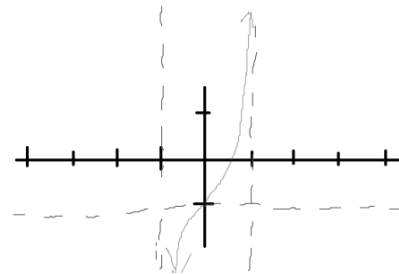
1. $y = \frac{1}{2} \cos \theta - 1$



2. $y = \csc x + 1$



3. $y = 2 \tan x - 1$



Rational Functions

End Behavior Asymptotes = Horizontal Asymptote

case 1: Highest Exponent in the denominator

$$y = \frac{1}{x}, \quad y = \frac{3x+4}{x^2-2}$$

E.B.A. $y=0$ $y=0$

case 2: Highest exponent in denominator and numerator

$$y = \frac{3x+2}{x-1}, \quad y = \frac{x^4-3x}{-4x^4}$$

E.B.A. $y=3$ $y=-\frac{1}{4}$ *Ratio of leading coeff.*

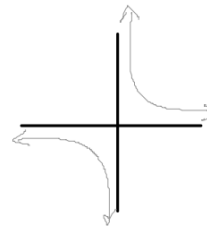
case 3: Highest exponent in numerator

$$y = \frac{5x^3+2x}{x^2-1}$$

E.B.A. $y=5x$

Divide the denominator into the numerator

$$f(x) = \frac{1}{x}$$



D: $(-\infty, 0) \cup (0, \infty)$

R: $(-\infty, 0) \cup (0, \infty)$

1. $f(x) = \frac{x+3}{x-4}$

VA: $x=4$

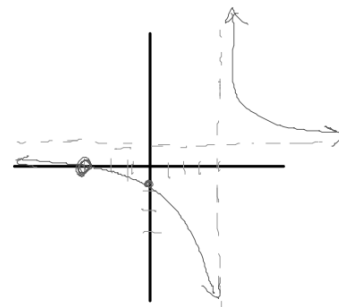
EBA (HA): $y=1$

x-int: $(-3, 0)$

y-int: $(0, -\frac{3}{4})$

D: $(-\infty, 4) \cup (4, \infty)$

R: $(-\infty, 1) \cup (1, \infty)$



2. $f(x) = \frac{-3}{x^2}$

VA: $x = 0$

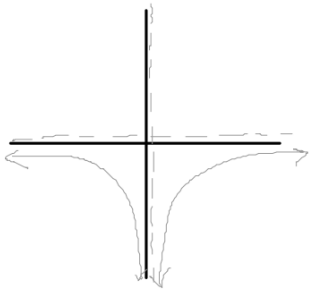
EBA (HA): $y = 0$

x-int: none

y-int: none

D: $(-\infty, 0)(0, \infty)$

R: $(-\infty, 0)$



3. $f(x) = \frac{x-5}{x^2-2x-8} = \frac{x-5}{(x-4)(x+2)}$

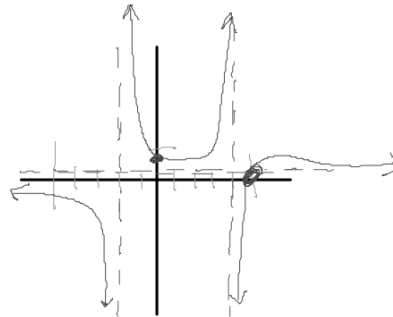
VA: $x = 4, -2$

EBA (HA): $y = 0$

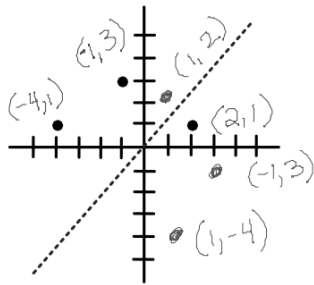
x-int: $(5, 0)$

y-int: $(0, \frac{5}{8})$

D: $(-\infty, -2)(-2, 4)(4, \infty)$



Inverse Functions Section 1.5



reflects over
 $y = x$

Find the inverse of $f(x)$.

1. $f(x) = 3x - 1$

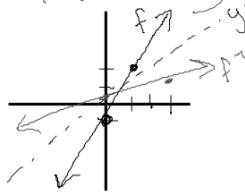
$$y = 3x - 1$$

$$\xrightarrow{f^{-1}} x = 3y - 1$$

$$x + 1 = 3y$$

$$\frac{x+1}{3} = y$$

$$f^{-1}(x) = \frac{x+1}{3}$$



2. $f(x) = x^3 + 6$

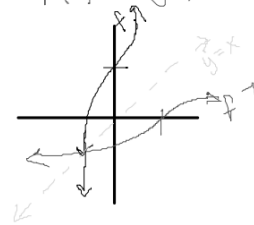
$$y = x^3 + 6$$

$$\xrightarrow{f^{-1}} x = y^3 + 6$$

$$x - 6 = y^3$$

$$\sqrt[3]{x-6} = y$$

$$f^{-1}(x) = \sqrt[3]{x-6}$$



Property of Inverse Functions:

$$f^{-1}(f(x)) = x$$

$$f(f^{-1}(x)) = x$$

3. Show that $f(x)$ and $g(x)$ are inverse functions algebraically.

$$f(x) = 3 - 4x \quad g(x) = \frac{3-x}{4}$$

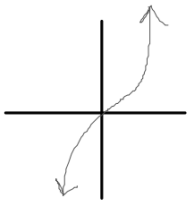
$$f(g(x)) = 3 - 4\left(\frac{3-x}{4}\right) \quad \text{and} \quad g(f(x)) = \frac{3 - (3-4x)}{4}$$

$$= 3 - (3-x) \quad = \frac{3-3+4x}{4}$$

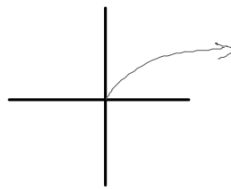
$$= x \quad = x$$

Inverse Functions must be "one-to-one" functions.

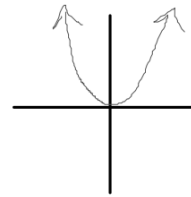
One-to-One functions must pass both the vertical and horizontal line tests



Yes - one to one fct.



Yes, one to one function



No, not one to one

4. Find the inverse of $f(x) = x^2 + 2$ for $x \leq 0$.

Why is there a condition? →

$$\begin{aligned} f^{-1} \quad y &= x^2 + 2 \\ x &= y^2 + 2 \end{aligned}$$

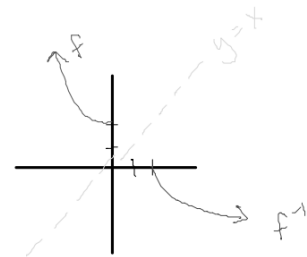
$$\sqrt{x-2} = y$$

$$f^{-1}(x) = -\sqrt{x-2}$$

negative to fit condition

$$\begin{aligned} f(x) \\ D: (-\infty, 0] \\ R: [2, \infty) \end{aligned}$$

$$\begin{aligned} f^{-1}(x) \\ D: [2, \infty) \\ R: (-\infty, 0] \end{aligned}$$



5. Determine if the function has an inverse. If so, find it.

$$f(x) = \sqrt{-x} - 5 \quad \leftarrow \text{Domain } (-\infty, 0]$$

$$y = \sqrt{-x} - 5$$

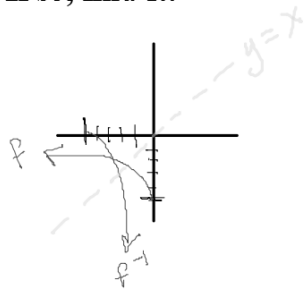
$$\begin{array}{l} f^{-1} \\ \hline x = \sqrt{-y} - 5 \end{array}$$

$$x + 5 = \sqrt{-y}$$

$$(x + 5)^2 = -y$$

$$-(x + 5)^2 = y$$

$$f^{-1}(x) = -(x + 5)^2$$



$f(x)$

D: $(-\infty, 0]$

R: $[-5, \infty)$

$f^{-1}(x)$

D: $[-5, \infty)$

R: $(-\infty, 0]$