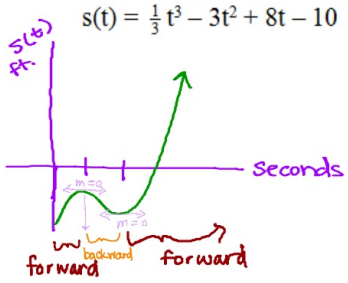
3. **A ball is rolling along the *x*-axis according to the function**

$s(t) = \frac{1}{3}t^3 - 3t^2 + 8t - 10$ *(ft.)*



$s'(t) = v(t)$

$s''(t) = v'(t) = a(t)$

a) When does the ball change direction?

$v(t) = 0 \qquad v(t) = t^2 - 6t + 8$

$t^2 - 6t + 8 = 0$

$(t-2)(t-4) = 0$

$t = 2, 4$ seconds

The ball changes direction at $t = 2$ and 4 seconds b/c $v(t)$ changes signs there.

b) When is the ball moving right?

The ball is moving right/forward on $[0,2)(4,\infty)$ b/c $v(t) > 0$

When is the ball moving left?

The ball is moving left/backward on $(2,4)$ b/c $v(t) < 0$

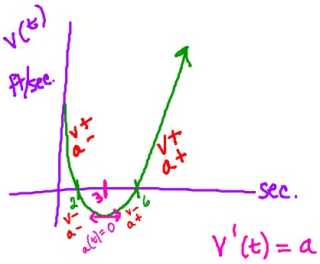
c) When is the ball accelerating?

speed up

The ball is speeding up on $(2,3)(4,\infty)$ b/c $v(t)$ and $a(t)$ are the same sign

slow down

The ball is slowing down on $[0,2)(3,6)$ b/c $v(t)$ and $a(t)$ are opposite signs



$v'(t) = a(t) = 2t - 6$

$0 = 2t - 6$

$2t = 6$

$t = 3$

d) Find the ball's displacement for the first 3 seconds.

displacement for the first 3 seconds $= s(3) - s(0)$

$= \left[\frac{1}{3}(3)^3 - 3(3)^2 + 8(3) - 10\right] - \left[\frac{1}{3}(0)^3 - 3(0)^2 + 8(0) - 10\right]$

$= [-4] - [-10]$

$= 6$ ft.

e) Average velocity of the ball for the first 3 seconds

Ave. velocity $= \frac{s(3) - s(0) \text{ ft.}}{3 - 0 \text{ sec}}$

$= \frac{6 \text{ ft}}{3 \text{ sec}}$

$= 2$ ft/sec.

f) Find the instantaneous velocity of the ball at t = 5 second

$s'(t) = v(t) = t^2 - 6t + 8$

$v(5) = (5)^2 - 6(5) + 8$

$v(5) = 3$ ft/sec

(Note: We can determine this is a reasonable answer b/c we know v(5) should be a positive value)

g) Find the acceleration of the ball at t = 1 second

$s''(t) = v'(t) = a(t) = 2t - 6$

$a(1) = 2(1) - 6$

$a(1) = -4$ ft/sec^2

4) A particle is moving along the y-axis according to the function

$y(t) = t^3 - 6t^2 + 7t - 3$ *(ft.)* ← up and down

a) Find the particle's *displacement* after 1 second?

$y(1) - y(0)$

$= (-1) - (-3)$

$= 2$ ft.

Find the particle's displacement after 2 seconds?

$y(2) - y(0)$

$= (-5) - (-3)$

$= -2$ ft.

Find the particle's displacement after 3 seconds?

$y(3) - y(0)$

$= (-9) - (-3)$

$= -6$ ft.

Note: Be sure to write $y'(t) = v(t) = 3t^2 - 12t + 7$ and $y''(t) = v'(t) = a(t) = 6t - 12$

b) Average velocity of the particle for the first 3 seconds

$$\frac{y(3) - y(0)}{3 - 0} = \frac{(-9) - (-3)}{3} = \frac{-6 \text{ ft}}{3 \text{ sec}} = -2 \text{ ft/sec}$$

c) Find the instantaneous velocity of the particle at t = 2

$y'(t) = v(t) = 3t^2 - 12t + 7$

$v(2) = 3(2)^2 - 12(2) + 7$

$v(2) = -5$ ft/sec

(Note: The speed is 5 ft/sec and the particle is moving downward)

d) Find the acceleration of the particle at t = 1

$y''(t) = v'(t) = a(t) = 6t - 12$

$a(1) = 6(1) - 12$

$a(1) = -6 \text{ ft/sec}^2$

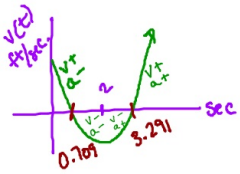
e) When does the particle change direction? (Note: Check places when $v(t) = 0$)

$y'(t) = v'(t) = 3t^2 - 12t + 7$

$0 = 3t^2 - 12t + 7$

$x = \dfrac{-(-12) \pm \sqrt{(-12)^2 - 4(3)(7)}}{2(3)}$

$x \approx 0.709$ and 3.291 seconds



f) When does the particle move up?

The particle moves up $[0, 0.709)(3.291, \infty)$ b/c $v(t) > 0$

When does the particle move down?

The particle moves down $(0.709, 3.291)$ b/c $v(t) < 0$

$v'(t) = a(t) = 6t - 12$
$0 = 6t - 12$
$6t = 12$
$t = 2$

g) When does the particle speed up?

The particle speeds up on $(0.709, 2)(3.291, \infty)$ b/c $v(t)$ and $a(t)$ are the same sign

When does the particle slow down?

The particle is slowing down on $[0, 0.709)(2, 3.291)$ b/c $v(t)$ and $a(t)$ are opposite signs

p. 135 #2

a) $A = \pi r^2$ $\quad C = 2\pi r$

$A = \pi\left(\dfrac{C}{2\pi}\right)^2$ $\quad \dfrac{C}{2\pi} = r$

$A = \dfrac{C^2 \pi}{4\pi^2}$

$A = \dfrac{C^2}{4\pi}$

b) $\dfrac{d}{dC}(A)$

$\dfrac{d}{dC}\left(\dfrac{C^2}{4\pi}\right)$

$\dfrac{d}{dC}\left(\dfrac{1}{4\pi}C^2\right)$

$= \dfrac{1}{2\pi}C$

$= \dfrac{C}{2\pi}$

c) $\dfrac{dA}{dC} = \dfrac{C}{2\pi}$

$\dfrac{dA}{dC}(\pi) = \dfrac{\pi}{2\pi} = \dfrac{1}{2}$

$\dfrac{dA}{dC} = \dfrac{in^2}{in}$