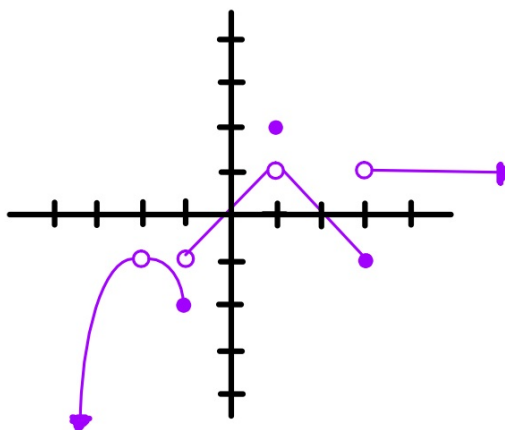


Limits and Rates of Change Section 2.1

Limits at a point:



$$\lim_{x \rightarrow 1^-} f(x) = 1$$

$$\lim_{x \rightarrow 1^+} f(x) = 1$$

$$\lim_{x \rightarrow 1} f(x) = 1$$

$$f(1) = 2$$

$$\lim_{x \rightarrow -2^-} f(x) = -1$$

$$\lim_{x \rightarrow -2^+} f(x) = -1$$

$$\lim_{x \rightarrow -2} f(x) = -1$$

$$f(-2) = \text{d.n.e.}$$

$$\lim_{x \rightarrow -1^-} f(x) = -2$$

$$\lim_{x \rightarrow -1^+} f(x) = -1$$

$$\lim_{x \rightarrow -1} f(x) = \text{d.n.e.}$$

$$f(-1) = -2$$

$$\lim_{x \rightarrow 3^-} f(x) = -1$$

$$\lim_{x \rightarrow 3^+} f(x) = 1$$

$$\lim_{x \rightarrow 3} f(x) = \text{d.n.e.}$$

$$f(3) = -1$$

Algebraically:

• plug-in the value

• Simplify the equation,
the plug-in the value

$$1. \lim_{x \rightarrow 3} (x+2) = (3)+2 \\ = \textcircled{5}$$

$$2. \lim_{x \rightarrow -3} (x^2 - x - 6) = (-3)^2 - (-3) - 6 \\ = 9 + 3 - 6 \\ = 6$$

$$3. \lim_{x \rightarrow 2} \frac{x}{x^2 - 4} = \frac{2}{(2)^2 - 4} \\ = \frac{2}{0} \textcircled{\text{d.n.e}}$$

$$4. \lim_{x \rightarrow 3} \frac{x^2 - 9}{x - 3} = \frac{(3)^2 - 9}{(3) - 3} = \frac{0}{0}$$

$$= \lim_{x \rightarrow 3} \frac{(x+3)\cancel{(x-3)}}{\cancel{x-3}}$$

$$= \lim_{x \rightarrow 3} (x+3)$$

$$= (3) + 3$$

$$= \textcircled{6}$$

$$5. \lim_{x \rightarrow 2} \frac{x-2}{x^2 - 4} =$$

$$= \lim_{x \rightarrow 2} \frac{\cancel{x-2}}{(x+2)\cancel{(x-2)}}$$

$$= \lim_{x \rightarrow 2} \frac{1}{x+2}$$

$$= \frac{1}{2+2}$$

$$= \textcircled{\frac{1}{4}}$$

Memorize

$\lim_{x \rightarrow 0} \frac{\sin x}{x} = 1$	$\sin(2x) = 2 \sin x \cos x$
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$$\begin{aligned} 6. \lim_{x \rightarrow 0} \frac{\sin(2x)}{x} &= \\ &= \lim_{x \rightarrow 0} \frac{2 \cdot \sin x \cdot \cos x}{x} \\ &= \lim_{x \rightarrow 0} 2 \cdot \frac{\sin x}{x} \cdot \cos x \\ &= 2 \cdot 1 \cdot \cos(0) \\ &= 2 \cdot 1 \cdot 1 \\ &= 2 \end{aligned}$$

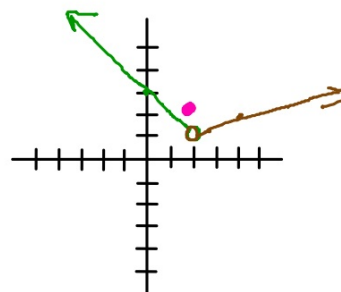
$$\begin{aligned} 7. \lim_{x \rightarrow 0} \frac{\sin x + x}{x} &= \\ &= \lim_{x \rightarrow 0} \frac{\sin x}{x} + \frac{x}{x} \\ &= 1 + 1 \\ &= 2 \end{aligned}$$

$$\begin{aligned} 8. \lim_{x \rightarrow \pi} \frac{\sin x + x}{x} &= \\ &= \lim_{x \rightarrow \pi} \frac{\sin x}{x} + \frac{x}{x} \\ &= \frac{\sin \pi}{\pi} + 1 \\ &= 1 \end{aligned}$$

p. 68: #58

$$c = 2 \quad f(x) = \begin{cases} 3-x, & x < 2 \\ 2, & x = 2 \\ \frac{x}{2} & x > 2 \end{cases}$$

a)



b) $\lim_{x \rightarrow 2^+} f(x) = 1$

$$\lim_{x \rightarrow 2^-} f(x) = 1$$

c) Yes,

$$\lim_{x \rightarrow 2^+} f(x) = \lim_{x \rightarrow 2^-} f(x)$$