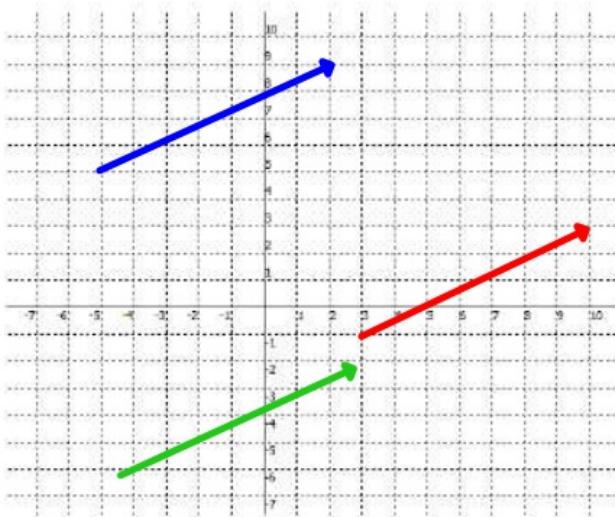


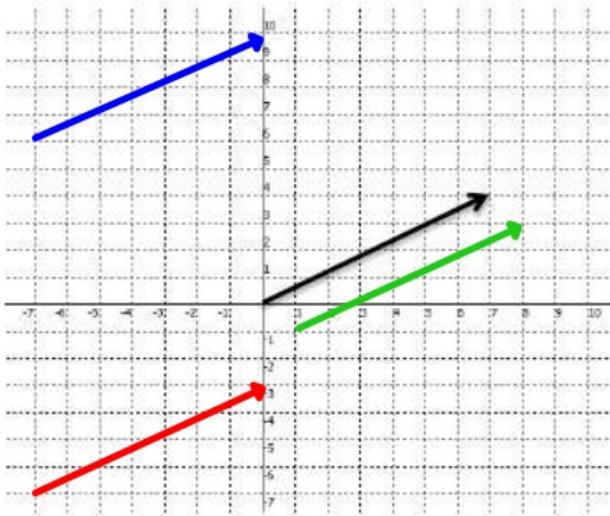
1. Draw the following segments using an arrow to indicate direction:

- from $(3, -1)$ to $(10, 3)$
- from $(-5, 5)$ to $(2, 9)$
- from $(-4.2, -6.1)$ to $(2.8, -2.1)$
- What do they have in common?



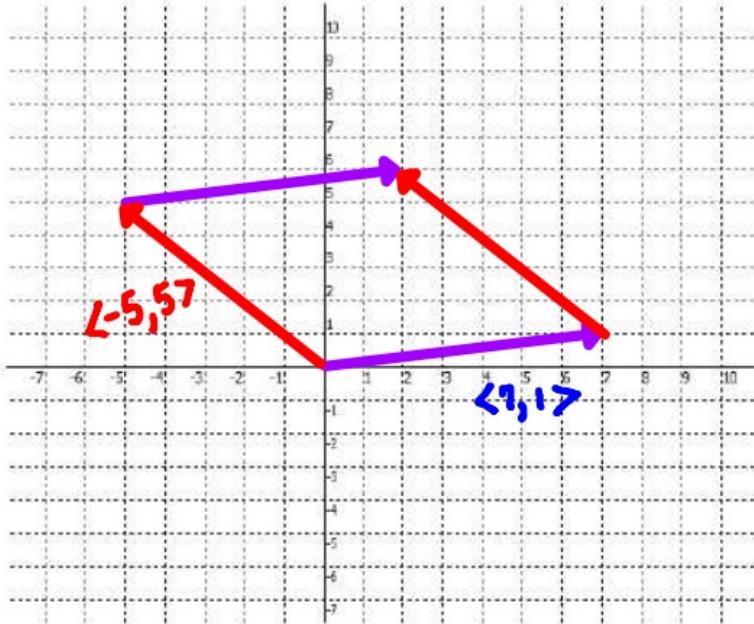
1.5 The diagram below shows vector $\vec{v} = \langle 7, 4 \rangle$ drawn in standard position.

- Draw 3 vectors equivalent to vector \vec{v} .



2. An equilateral quadrilateral is called a *rhombus*.

- Give coordinates of a rhombus whose diagonals and sides are *not* parallel to the rulings on your graph paper.
- Explain the process you used to generate your example.

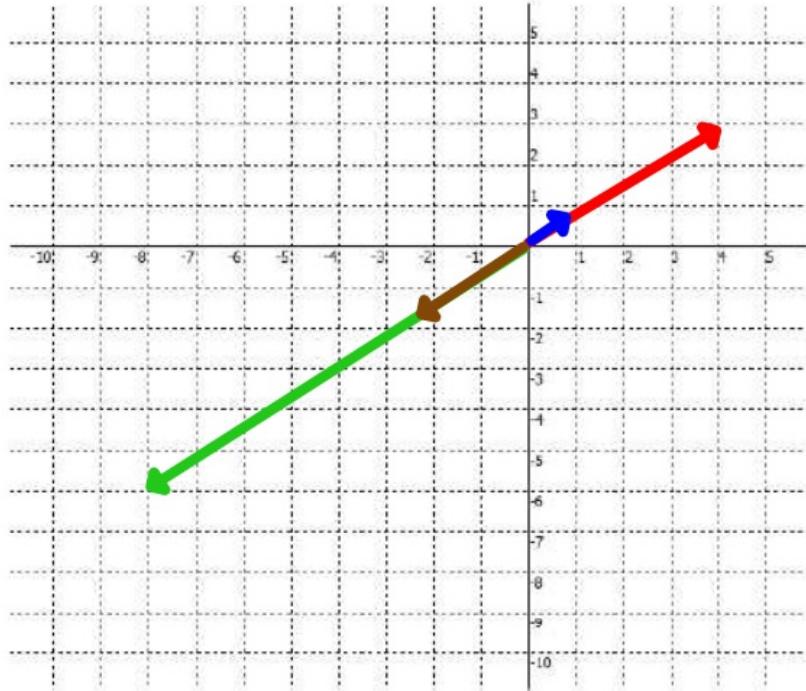


$$\langle -5, 5 \rangle \quad |\langle -5, 5 \rangle| = \sqrt{(-5)^2 + (5)^2} = \sqrt{50}$$

$$\langle 7, 1 \rangle \quad |\langle 7, 1 \rangle| = \sqrt{(7)^2 + (1)^2} = \sqrt{50}$$

3. Consider the vector $\vec{v} = \langle 4, 3 \rangle$

SCALING VECTORS



a. Find the length of the vector \vec{v} . $\sqrt{(4)^2 + (3)^2} = \sqrt{25} = 5$

b. Sketch a vector that points in the opposite direction and has length 10.

$$\begin{aligned} 5(2) &= 10 \\ 3(-2) &= 6 \\ 4(2) &= 8 \end{aligned}$$

$$\langle -8, 6 \rangle$$

$$\sqrt{(-8)^2 + (6)^2} = \sqrt{64+36} = 10$$

c. Sketch a vector that points in the same direction and has length 1.

$$\begin{aligned} 5(\frac{1}{5}) &= 1 \\ 3(\frac{1}{5}) &= \frac{3}{5} \\ 4(\frac{1}{5}) &= \frac{4}{5} \end{aligned}$$

$$\langle \frac{4}{5}, \frac{3}{5} \rangle$$

$$\sqrt{(\frac{4}{5})^2 + (\frac{3}{5})^2} = \sqrt{\frac{16}{25} + \frac{9}{25}} = 1$$

d. Sketch a vector that points in the opposite direction with length 3.

$$\begin{aligned} 5(\frac{3}{5}) &= 3 \\ 3(\frac{3}{5}) &= \frac{9}{5} \\ 4(\frac{3}{5}) &= \frac{12}{5} \end{aligned}$$

$$\langle -\frac{12}{5}, \frac{9}{5} \rangle$$

$$\sqrt{(-\frac{12}{5})^2 + (\frac{9}{5})^2} = \sqrt{\frac{144}{25} + \frac{81}{25}} = \sqrt{\frac{225}{25}} = 3$$

4. Plot $A = (-1, 3)$ and $B = (7, -3)$, and draw the line \overline{AB} .



a. Use a vector to find the midpoint of segment \overline{AB} , that is, the point half-way from A to B .

$$\begin{aligned} & A + \frac{1}{2} \overrightarrow{AB} \\ &= (-1, 3) + \frac{1}{2} \langle 8, -6 \rangle \\ &= (-1, 3) + \langle 4, -3 \rangle \\ &= (3, 0) \end{aligned}$$

b. Now use your method to find the point P that is one-third of the way from A to B .

$$\begin{aligned} & A + \frac{1}{3} \overrightarrow{AB} \\ &= (-1, 3) + \frac{1}{3} \langle 8, -6 \rangle \\ &= (-1, 3) + \left\langle \frac{8}{3}, -2 \right\rangle \\ &= \left(-1 + \frac{8}{3} \right), 3 - 2 \rangle \\ &= \left(\frac{5}{3}, 1 \right) \end{aligned}$$

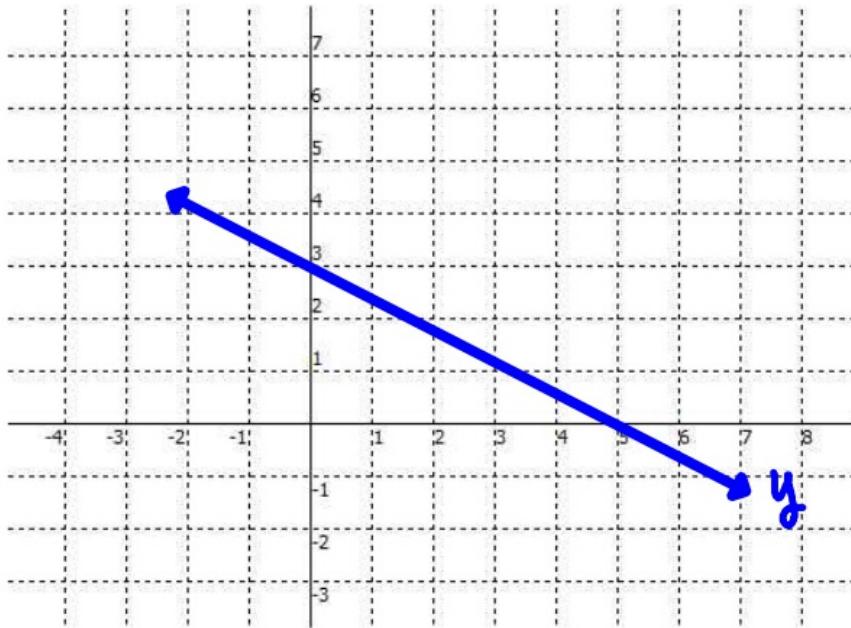
c. Find a point on line \overline{AB} that is 6 units away from B .

$$|\langle 8, -6 \rangle| = 10 \quad \text{I need a vector of length 6}$$

$$\begin{aligned} & \frac{6}{10} \overrightarrow{AB} \text{ and apply to point } B \\ & B + \frac{6}{10} \overrightarrow{AB} \text{ or } (7, -3) + \frac{6}{10} \langle 8, -6 \rangle \\ &= (7, -3) + \langle 4.8, -3.6 \rangle \\ &= (11.8, -6.6) \end{aligned}$$

$$y = -\frac{3}{5}x + 3$$

5. Graph the line $3x + 5y = 15$.



a. Find the direction vector of the line

$$\text{Slope } m = -\frac{3}{5}$$

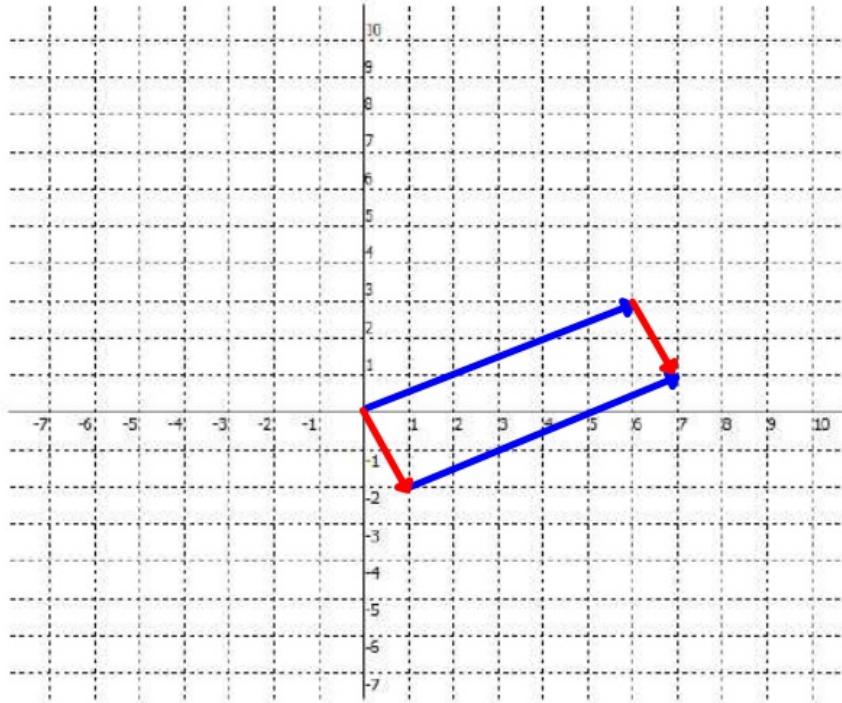
Expressed as a vector $\langle 5, -3 \rangle$

b. What vector(s) are perpendicular to $3x + 5y = 15$?

$$\perp m = \frac{5}{3}$$

expressed as a vector $\langle 3, 5 \rangle$

6. Find coordinates for the vertices of a *lattice rectangle* that is three times as long as it is wide, with none of the sides horizontal.



Start with vector $\langle 1, -2 \rangle$

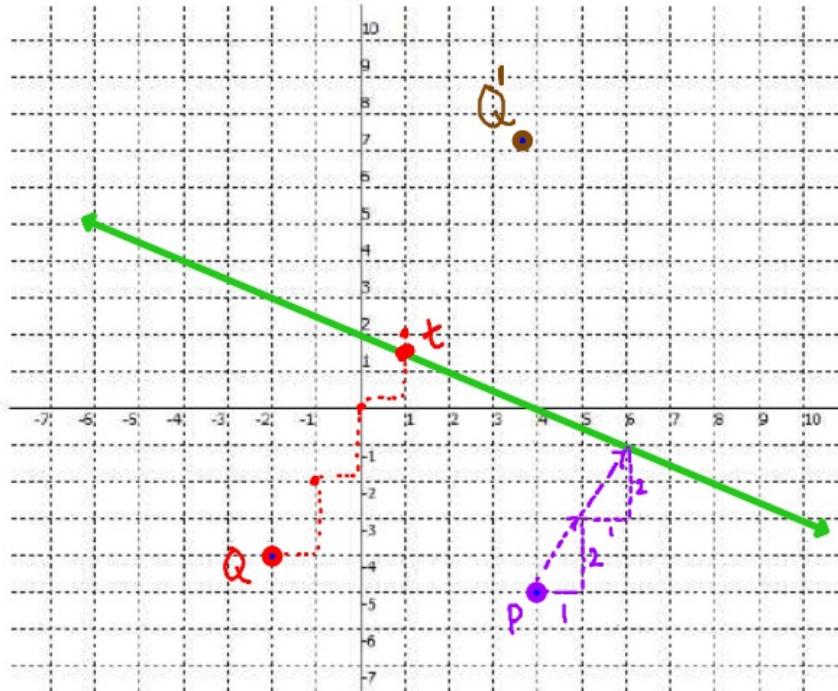
I need a Rt $\frac{\pi}{4}$: So need a \perp vector: $\langle 2, 1 \rangle$

then triple that: $\langle 6, 3 \rangle$

Use equivalent vectors to complete the lattice rectangle

$$y = -\frac{1}{2}x + 2$$

7. Graph the line $x + 2y = 4$



a. Find the distance from the point $P = (4, -5)$ to the line $x + 2y = 4$.

Vector \perp to the line $\langle 1, 2 \rangle$
 $2 \langle 1, 2 \rangle$

$$d = |2 \langle 1, 2 \rangle|$$

$$d = 2\sqrt{5}$$

b. Find the distance from the point $Q = (-2, -4)$ to the line $x + 2y = 4$.

$$(-2, -4) + t \langle 1, 2 \rangle = (-2+t, -4+2t)$$

$$= (x, y) \text{ a point on the line}$$

$$\begin{aligned} x + 2y &= 4 \\ (-2+t) + 2(-4+2t) &= 4 \\ -2+t-8+4t &= 4 \\ t &= \frac{14}{5} \end{aligned}$$

Vector to the line:
 $\left| \frac{14}{5} \langle 1, 2 \rangle \right|$
 $\frac{14}{5} \cdot \sqrt{5}$

c. Find Q' , the reflection of Q across the line $x + 2y = 4$.

to the line: $t = \frac{14}{5}$, so reflection: $2 \left(\frac{14}{5} \right)$

$$\begin{aligned} &(-2, -4) + \frac{28}{5} \langle 1, 2 \rangle \\ &= \left(-2 + \frac{28}{5}, -4 + \frac{56}{5} \right) \\ &= (3.6, 7.2) \end{aligned}$$

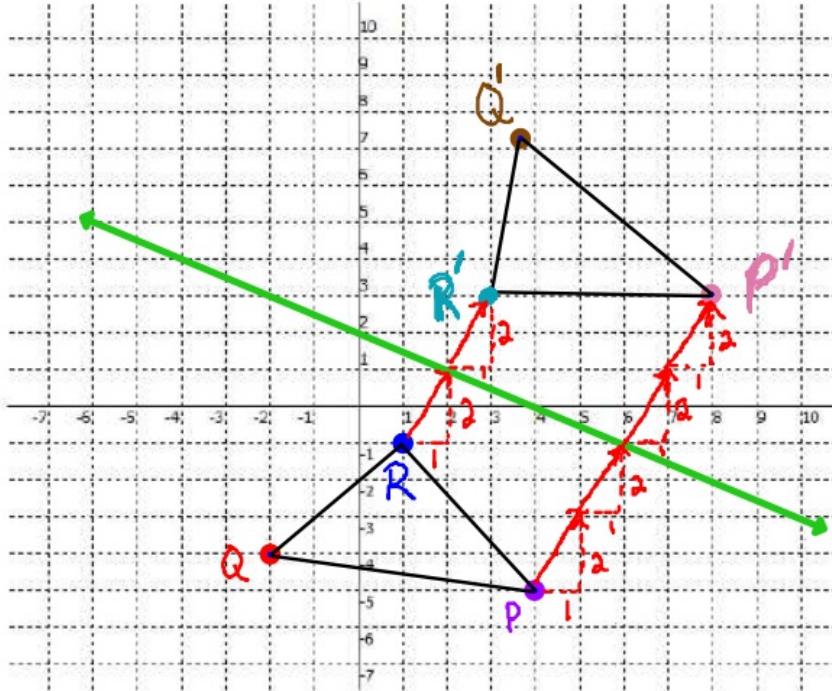
d. Let $R = (1, -1)$; draw $\triangle PQR$, then find $\triangle P'Q'R'$, the reflection across the line $x + 2y = 4$.

See the next page

Continued from previous page

$$\rightarrow y = -\frac{1}{2}x + 2$$

7. Graph the line $x + 2y = 4$



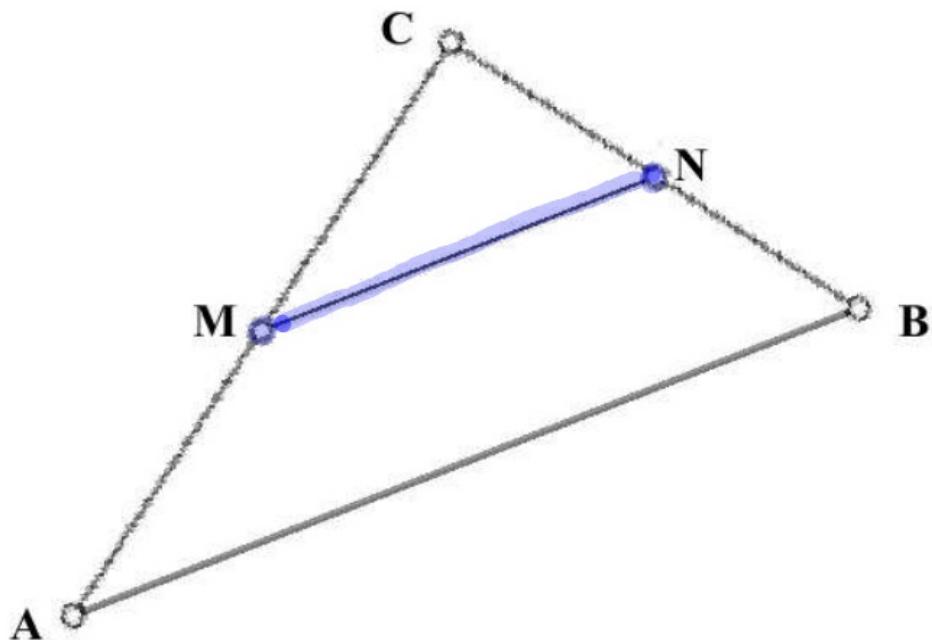
d. Let $R = (1, -1)$; draw $\triangle PQR$, then find $\triangle P'Q'R'$, the reflection across the line $x + 2y = 4$.

$$R' = (1, -1) + 2 \langle 1, 2 \rangle = (3, 3)$$

$$P' = (4, -5) + 4 \langle 1, 2 \rangle = (8, 3)$$

MIDLINES & COORDINATE PROOF

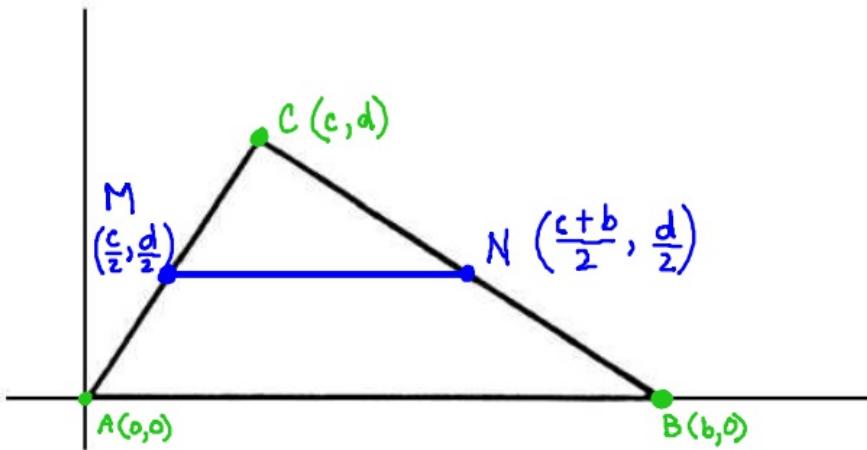
Definition: The midline of a triangle is a segment whose endpoints are the midpoints of the sides of the triangle.



In the diagram shown, M and N are midpoints of sides \overline{AC} and \overline{BC} respectively, the segment \overline{MN} is a midline of $\triangle ABC$.

Use vectors to show that the midline is parallel to *and* half the length of the base of the triangle.

8. In $\triangle ABC$, M & N are midpoints of sides \overline{AB} and \overline{BC} respectively;
the segment \overline{MN} is a midline of $\triangle ABC$.



- a. Use vectors to show that the midline is parallel to the base of the triangle.

$$\mathbf{m}_{AB} = \frac{\mathbf{O}-\mathbf{O}}{b-0} = \frac{0}{b} = 0 \quad \mathbf{m}_{MN} = \frac{\frac{d}{2}-\frac{d}{2}}{\frac{c+b}{2}-\frac{c}{2}} = \frac{0}{\frac{b}{2}} = 0$$

- b. Verify the midline segment is half the length of the base of the triangle.

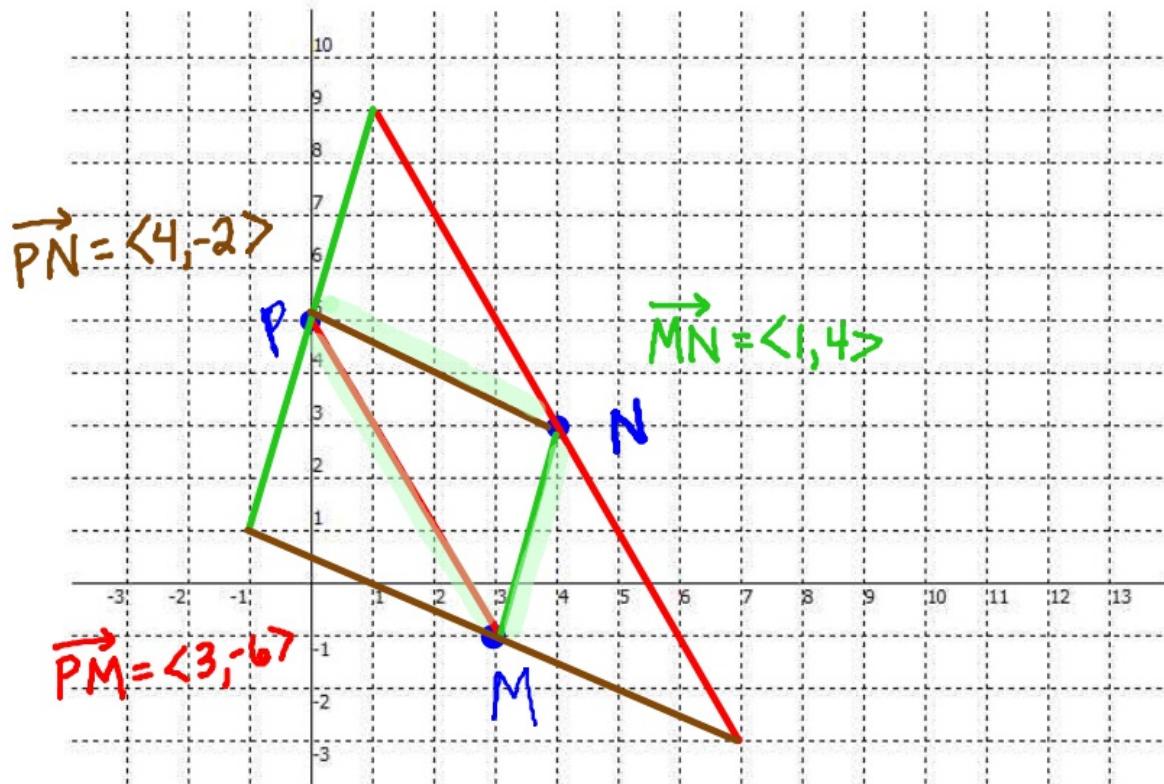
$$\begin{aligned}\overrightarrow{AB} &= \langle b, 0 \rangle & \overrightarrow{MN} &= \left\langle \frac{b+c}{2} - \frac{c}{2}, \frac{d}{2} - \frac{d}{2} \right\rangle \\ & & &= \left\langle \frac{b}{2}, 0 \right\rangle \\ & & &= \frac{1}{2} \langle b, 0 \rangle \\ & & &= \frac{1}{2} \overrightarrow{AB}\end{aligned}$$

M N P

9. The midpoints of the sides of a triangle are $(3, -1)$, $(4, 3)$, and $(0, 5)$.

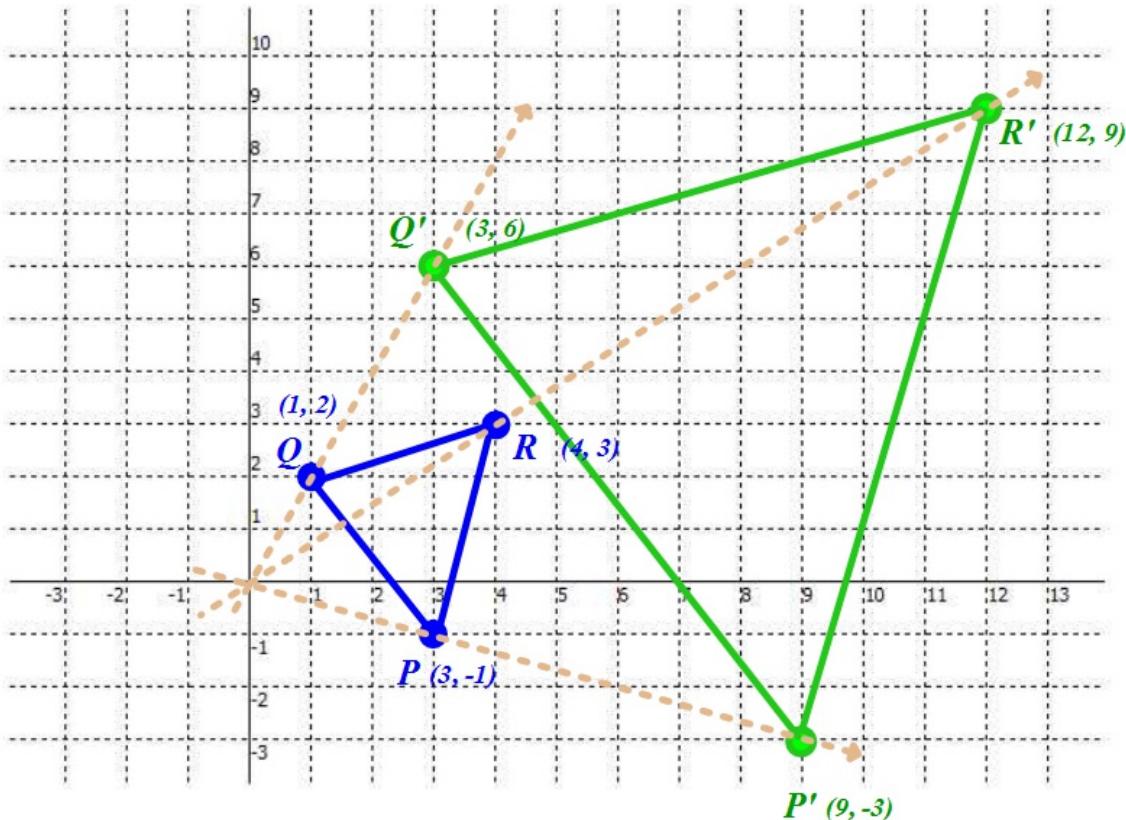
- Find coordinates for the vertices of the triangle.

(Hint: All of these segments are midlines – use vectors!)



Coordinates of triangle
 $(7, 9)$ $(-1, 1)$ $(0, 5)$

10. Plot the points $P=(3, -1)$, $Q=(1, 2)$ and $R=(4, 3)$, and draw $\triangle PQR$.



- Apply the transformation $T(x, y) = (3x, 3y)$ to the coordinates of the points P , Q , and R to form a new triangle with vertices P' , Q' , and R' . Plot the image.
- Compare the sides and angles of the image triangle $\triangle P'Q'R'$ with the corresponding parts of $\triangle PQR$.
- Draw lines that connect each point to its image, and extend those lines until they intersect at $C = (0,0)$, the dilation center.
- What is the relationship between $\overrightarrow{Q'P'}$ and \overrightarrow{QP} .

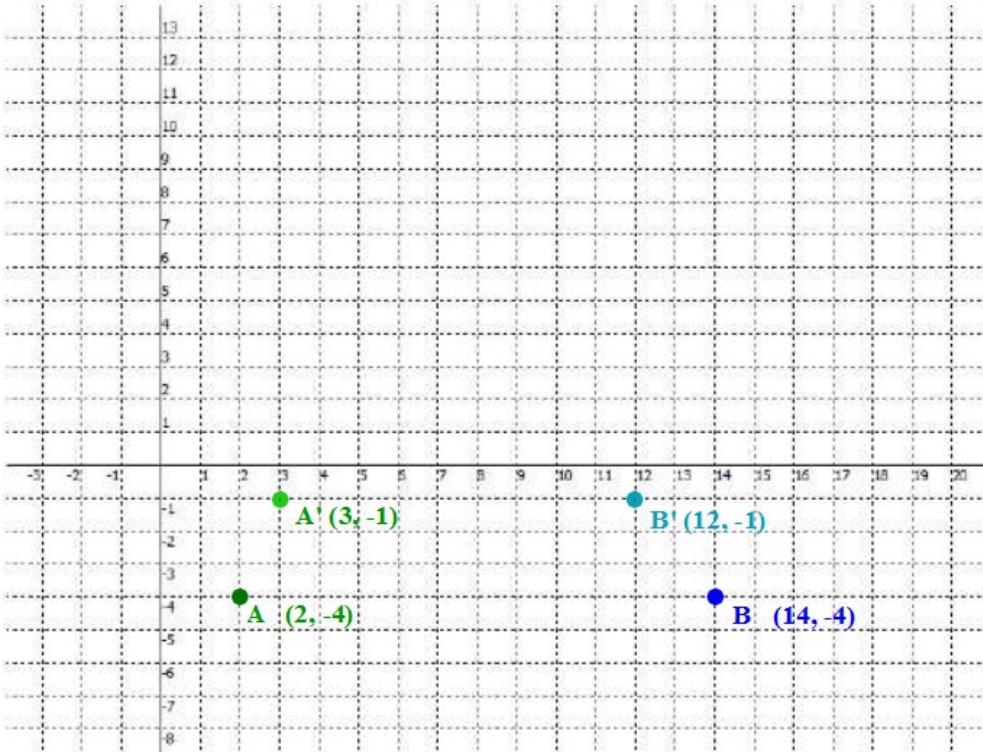
$$\overrightarrow{Q'P'} = \langle 6, -9 \rangle = 3\langle 2, -3 \rangle = 3\overrightarrow{QP}$$

- What is the relationship between $\overrightarrow{CQ'}$ and \overrightarrow{CQ} .

$$\overrightarrow{CQ'} = \langle 3, 6 \rangle = 3\langle 1, 2 \rangle = 3\overrightarrow{CQ}$$

#11 Answers given at the conference on next page

11. A mystery transformation sends $A = (2, -4)$ to $A' = (3, -1)$ and $B = (14, -4)$ to $B' = (12, -1)$.



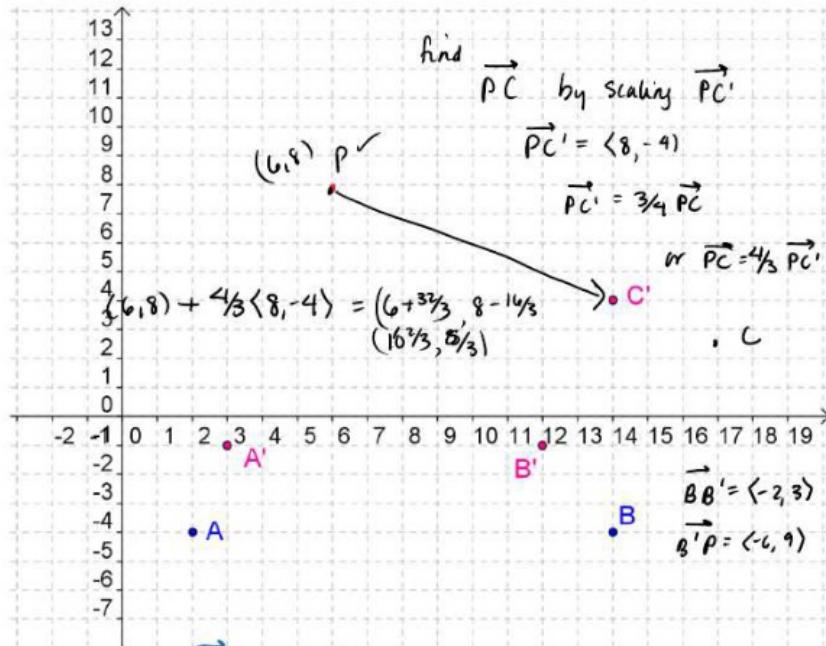
a. If you know this is a similarity transformation, find the coordinates of the dilation center P.

b. If $C' = (14, 4)$, find the coordinates of C , the pre-image of C' .

#11 Answer from conference on this page

A mystery transformation sends $A = (2, -4)$ to $A' = (3, -1)$ and $B = (14, -4)$ to $B' = (12, -1)$.

- If you know this is a similarity transformation, find the coordinates of the dilation center P.
- If $C' = (14, 4)$, find the coordinates of C, the pre-image of C' .



Contraction

$\overrightarrow{AB} = \langle 12, 0 \rangle$ $\frac{a}{b} = \frac{3}{4}$
 $\overrightarrow{A'B'} = \langle 9, 0 \rangle$ $\frac{3}{4} \overrightarrow{AB} = \overrightarrow{A'B'}$

Dilation center: P

$\frac{3}{4} \overrightarrow{PA} = \overrightarrow{PA}' \quad \text{or} \quad \frac{\overrightarrow{PA}'}{\overrightarrow{PA}} = \frac{3}{4}$

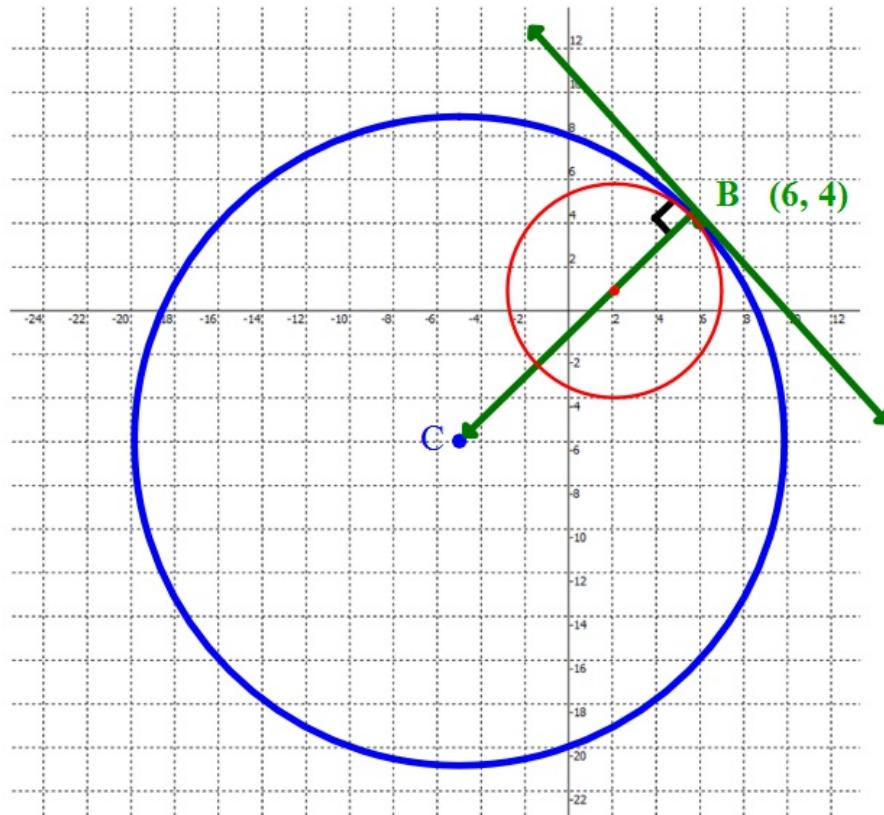
$\overrightarrow{PA} : \overrightarrow{PA}' = 3 : 4$

$\overrightarrow{AA'} = \langle +1, 3 \rangle$

$\overrightarrow{A'P} = 3 \langle +1, 3 \rangle = \langle +3, 9 \rangle \rightarrow$ take me to P

$\overrightarrow{PA} : \overrightarrow{PA}' = 3 : 4$

12. Sketch the circle C with the equation $(x + 6)^2 + (y + 5)^2 = 225$



a. State the center and radius of $\odot C$

$$C = (-6, -5) \text{ and } r = 15$$

b. Find the equation of the circle with radius 5 which is *internally tangent* to the given circle at the point (6,4).

$$\vec{BC} = \langle -12, -9 \rangle$$

$$|\vec{BC}| = 15$$

To find the center of new circle
 $\frac{1}{3}\vec{BC}$ and apply to B

$$\frac{1}{3}\langle -12, -9 \rangle + (6, 4)$$

$$= \langle -4, -3 \rangle + (6, 4)$$

$$= (2, 1) \text{ with a radius of } 5$$

$$(x-2)^2 + (y-1)^2 = 25$$

c. Find the equation of their common tangent.

point
 $(6, 4)$

slope
 $\perp \vec{BC} =$

$$m_{BC} = \frac{-9}{12} = \frac{3}{4}$$

$$\perp m_{BC} = -\frac{4}{3}$$

$$y = mx + b$$

$$4 = -\frac{4}{3}(6) + b$$

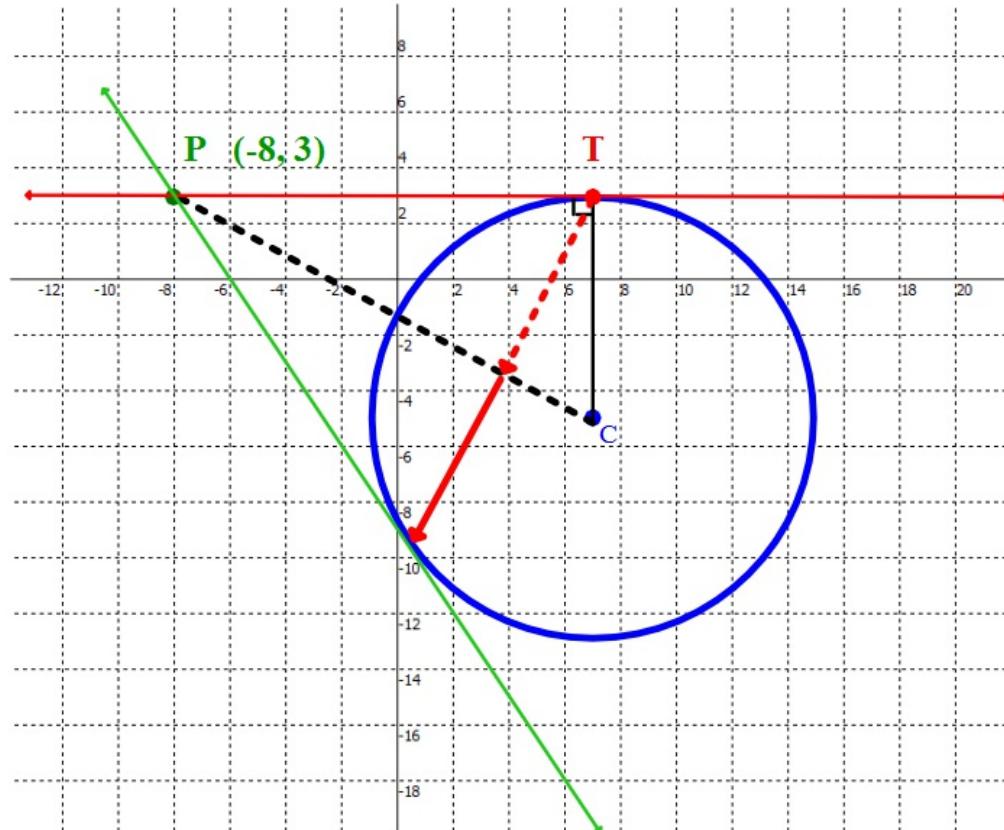
$$4 = -8 + b$$

$$12 = b$$

$$y = -\frac{4}{3}x + 12$$

**Here is a beginning to
finding the solutions**

13. Sketch the circle C with equation $(x-7)^2 + (y+5)^2 = 64$



- a. Find the length of the tangent segments from the point $P=(-8, 3)$ to the circle with equation $(x-7)^2 + (y+5)^2 = 64$.

- b. Find the equation of both tangent lines and both points of tangency.
(Use a vector to find the 2nd point of tangency.)

Credit

A big thanks to Amanda A. Simmons who presented V is for Vector at the NCTM National Conference 2012
This document is a slightly revised form of her presentation.

Resources for additional problems:

- Exeter mathematics Department: www.exeter.edu
Look for Teaching Materials under the Mathematics Dept. page: MATH 2
- “old fashioned” texts in Analytic Geometry
- the coordinate geometry section of a more traditional geometry text